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XX. Experimental researches on the strength of pillars of cast iron, and other materials

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XX. *Experimental Researches on the Strength of Pillars of Cast Iron, and other Materials. By* EATON HODGKINSON, *Esq. Communicated by* PETER BARLOW, *Esq., F.R.S., &c.*

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WHEN we consider to what extent pillars of iron and of timber are used for the support of buildings, and reflect that there are no satisfactory rules by which to measure the strength of pillars, it becomes a matter of great importance to obtain such rules by means of experiment, and, if possible, to discover the laws on which they are founded. A feeling of this kind, heightened by the remarks of Dr. ROBISON, in his Mechanical Philosophy, and by the strongly-expressed opinion as to our want of such knowledge by Mr. BARLOW*, led me to wish to undertake the inquiry. I mentioned the matter, therefore, to my friend Mr. FAIRBAIRN, who, with that liberality which I have experienced from him on former occasions, at once put every means of a full investigation into my hands. He expressed a wish that I should extend the inquiry to pillars of various kinds, ancient as well as modern; and leave no part of the subject in uncertainty for want of experiments sufficiently varied and extensive. Thus freed from restraint, I endeavoured, in my wish to acquire the requisite information, to forget the expense to which I put my friend, and have made every effort to render the experiments at least ample, correct, and useful. The pillars on which the experiments were made, were mostly of cast iron, as being the material in most general use; but some were of wrought iron and steel, and others of wood.

In the earlier experiments, the pillars used were uniform cylinders, either with their ends rounded, so that the crushing force might pass through the axis, or with flat and parallel ends, so that the pillar, when placed between two perfectly parallel crushing surfaces, might have its ends completely bedded against them.

2. The pillars during the experiments were placed vertically, resting upon a flat, smooth plate of hardened steel, laid upon a cast iron shelf, made very strong, and lying horizontal. The pressure was communicated to the upper end of the pillar by means of a strong lever acting upon a bolt of hardened steel, two and a half inches diameter, and about a foot long, kept vertical by being made to pass through a hole bored in a deep mass of cast iron; the hole being so turned as just to let the bolt slide easily through without lateral play. The top of the bolt was hemispherical, that the pressure from the lever might act through its axis; and the bottom was turned

* Report to the British Association for the Advancement of Science.

flat to rest upon the pillar. The bottom of this bolt, and the shelf on which the pillar stood, were necessarily kept parallel to each other; for the mass through which the bolt passed, and that on which the shelf rested, were parts of the same large case of iron, cast in one piece, and so formed as to admit shelves at various heights, for breaking pillars of different lengths. The case had three of its four sides closed; circular apertures were, however, made through them, that the experimenter might observe the pillar without danger.

Fig. 1, Plate XIII., will show more clearly the nature of the apparatus, and the manner in which the experiments were made. Thus A B is the lever, about fourteen feet long, generally acting with a leverage of from four to seven; C D is an iron frame-work, in which the fixed end of the lever is placed, and in which it turns as on a pivot, the frame C D being fixed firmly in a strong heavy wall E F, part of which is imagined in the drawing to have been removed, to show the fixings and apparatus; G is the steel bolt on which the lever acts; H the socket through which it passes; H I K the iron case admitting shelves as P, at different heights, for pillars of different lengths not exceeding five feet; L M an additional part to be attached to the box H I K, for pillars of greater length; N O is one of the pillars resting upon the shelf P, and pressed upon by the pin G; Q is the weight, which could be removed to any part of the lever, and increased or diminished at pleasure; or entirely taken off the pillar, at any time, by means of the winding apparatus.

Experiments.

3. In order to ascertain the laws connecting the strength of cast iron pillars with their dimensions, they were broken of various lengths, from five feet to one inch; and the diameters varied from half an inch to two inches, in solid pillars; and in the hollow ones, the length was increased to seven feet six inches, and the diameter to three inches and a half. My first object was to supply the deficiencies of EULER's theory of the strength of pillars*, if it should appear capable of being rendered practically useful; and, if not, to endeavour to adapt the experiments so as to lead to useful results.

4. As the results of the experiments were intended to be compared together, it was desirable that all the pillars of cast iron should be from one species of metal; and the description chosen was a Yorkshire iron, the Low Moor, No. 3. The pillars were mostly made cylindrical, as that seemed a more convenient form in experiments of this kind than the square; for square pillars generally break anglewise. The experiments, in the first table annexed, were made on solid uniform pillars, rounded at the ends, that the force might pass along the axis; and the metal was cast in dry sand, to obtain, as far as possible, uniformity in its texture. In the second table, the pillars were uniform and cylindrical, as before, but had their ends flat and at right angles to the axis. The pillars were from the same models as before, but were cast

* Berlin Memoirs, 1757, Petersburgh Commentaries, 1778.

in green (moist) sand, which rendered the operation of casting less troublesome; and it was conceived that the mode of casting would make little or no difference in the strength, a conjecture which was proved afterwards to be correct. I have not given here an explanation of these or the other tables, as it is hoped they will be understood by inspection. The forms of the fractures of many of the pillars, particularly those in Tables I. and II., are given in conjunction with the drawings of the pillars in Plate XV. The position of the neutral line was sought for with great care, and is inserted in the tables wherever it could be obtained with any degree of certainty.

Conclusions from the First and Second Tables.

5. The experiments in the second table were undertaken after those in the first were nearly completed; for I found, by a trial or two, that a pillar with flat ends was much stronger than one of the same diameter and length with its ends rounded. To ascertain, therefore, the ratio of the strength in the two cases, and whether it was constant under all changes of dimension, it became necessary to make the second table to include most of the lengths and diameters of the former; and, indeed, other considerations made me reduce the lengths in it, till the pillars broke by crushing without flexure.

Taking, then, the mean breaking weights of pillars from the same models in the first two tables of experiments, and neglecting the small differences in the measured diameters of the castings, we have the ratio of the strengths as in the following abstract.

Abstract from Tables I. and II., &c.

Pillars with rounded ends.				Pillars with flat ends.				Ratio of breaking weights of pillars with rounded and flat ends, the diameters being considered equal.
Length.	Mean diameter.	Number of experiments deduced from.	Breaking weight.	Mean diameter.	Number of experiments deduced from.	Breaking weight.	Weight which would crush the cylinder.	
inches.	inch.		lbs.	inch.		lbs.	lbs.	
60·5	·50	2	143	·51	2	487*	22430	1 : 3·405
60·5	·77	2	780	·77	2	2456	51130	1 : 3·148
60·5	·99	2	1902	·997	3	6238	85721	1 : 3·279
60·5	1·29	2	5707	1·29	3	16064	143508	1 : 2·815
60·5	1·52	2	10861	1·56	3	28962	209869	1 : 2·667 } Mean Ratio 1 : 3·167
30·25	·50	3	539	·50	1	1662	21559	1 : 3·083
30·25	·77	1	2726	·77	3	8811	51130	1 : 3·232
30·25	·99	2	6105	1·01	4	20310	87971	1 : 3·326
15·125	·50	3	1904	·51	3	6764	22430	1 : 3·552
20·166	·767	2	6602	·777	3	15581	52064	1 : 2·3600
20·166	1·01	2	15737	1·022	2	31804	90074	1 : 2·0210
15·125	·76	3	9223	·775	4	21059	51797	1 : 2·2833
7·5625	·497	3	5262	·50	3	11255	21559	1 : 2·1389
15·125	·99	3	19752	1·000	2	40250	86238	1 : 2·0378
10·083	·76	3	17506	·768	4	25923	50865	1 : 1·4808
7·5625	·77	2	22948	·777	3	32007	52064	1 : 1·3947
3·7812	·50	2	15107	·52	3	24616	23319	1 : 1·6294

* This pillar had discs on the ends.

6. The preceding abstract contains the results of experiments with pillars varying in length, from 121 times the diameter, down to $7\frac{1}{2}$ times; and we see, from the last

column, that the ratio of the strengths of pillars, with rounded and with flat ends, is nearly constant for all pillars, from those which are the longest compared with their diameters, to those whose length is about thirty times the diameter; which is the case with the pillars $30\frac{1}{4}$ inches long, and 1·01 inch diameter; and also those of 15·125 inches long and ·51 inch diameter. In all pillars, therefore, whose length is thirty times the diameter, or upwards, the strength of those with flat ends seems to be about three times as great as the strength of those of the same dimensions with rounded ends, the mean ratio being 1 : 3·167. In the pillars shorter than thirty times the diameter, the ratio is not constant: and it will be seen that it decreases when the length compared with the diameter decreases. This uniformity in all pillars down to a certain length, compared with their diameter, and the departure from it in shorter pillars, were matters which for a considerable time arrested my attention. Experiments were made upon wood and wrought iron, and the results were in accordance with these; but the reason of the deviation was first made evident by the experiments upon wrought iron, from which it appeared that short specimens of that metal became slightly crushed, and perceptibly changed in their form, by a weight which was nearly the same as that which produced a falling off in the strength of a long pillar of the same diameter. Thus it was found that a weight of 22679 lbs. produced a very obvious change in the form of a short cylinder, 1·015 inch diameter (as will be seen from the experiments further on): and a pillar of that diameter and $30\frac{1}{4}$ inches long, with the ends flat, exhibited a great falling off in strength. It was evident, therefore, that incipient crushing or derangement of the parts was the cause of the change. Cast iron does not often admit of such an alteration in its form as to be perceptible to the eye; but its strength begins to fall off with weights which differ not widely from those which deranged the form, and reduced the strength of wrought iron. I have, therefore, made the preceding abstract to include, in addition to the mean breaking weights from the first and second tables, a column containing the weights which would crush, without flexure, short cylinders of cast iron (the Low Moor, No. 3.) of the same diameters as the pillars with flat ends. These crushing weights were obtained from the experiments further on (Art. 55.), in which it was shown that prisms of this iron, perfectly bedded at the ends, and so short as not to be bent with the pressure, were crushed with a force of about forty-nine tons per square inch. The mean of my experiments gave 86238 lbs. for the crushing force of a cylinder one inch diameter; and the resistance to crushing, in the cylinders of other diameters, was calculated from this, on the supposition that the resistance is as the area of the section; which supposition was proved to be true in my Report on the strength of hot and cold blast iron*.

7. It will be seen at a glance from the preceding abstract, what portion of its crushing strength each pillar required to break it, whether the ends were rounded or flat. The pillars with flat ends, whose length was the least number of times the

* Transactions of the British Association for the Advancement of Science, vol. vi.

diameter (the ratio of the strengths of the pillars with rounded and with flat ends being uniform), required $\frac{6764}{22430}$ and $\frac{20310}{87971}$ of the crushing weights to break them. The lengths of these pillars were thirty times their diameters, which were ·51 and 1·01 inch respectively; but the pillars ·777 inch diameter, and 20·166 inches long, (the length being twenty-four times the diameter), were broken with $\frac{15581}{52064}$ of the crushing weight. These last pillars, which were broken with a little less than one-third of the crushing weight, showed themselves injured by the compression, since they gave a ratio less than the mean. On the other hand, the pillars just mentioned as ·51 inch diameter were broken with from one-fourth to one-third of the crushing weight; and those 1·01 inch diameter with somewhat less than one-fourth, neither of these being injured by the pressure. We may, therefore, conclude that about one-fourth of the crushing weight is the greatest load which a cast iron pillar, flat at the ends, will bear without producing a crushing or derangement of the materials, which would lessen its breaking weight; and that the length of such a pillar should be thirty times the diameter, or upwards. Pillars whose length is less than in this proportion, give the ratio of the strengths of those with rounded and with flat ends, from 1 : 3, down to 1 : $1\frac{1}{2}$, or less, according as we reduce the number of times which the length exceeds the diameter, as will be seen by the abstract.

A circumstance may be mentioned, which, perhaps, tended a little to equalize the strengths of the shortest pillars mentioned above. It became necessary to render those which were rounded at the ends more flat there, than if the ends had been hemispheres; whilst in the experiments upon pillars, whose length was greater with respect to the diameter than these, the ends were more prominent than in the hemi-spherical form. This change became necessary on account of the splitting of the ends in the short pillars; it having been found that the pillars whose diameter was one-thirteenth of the length or upwards, with rounded ends, failed in many instances by the ends becoming split. In these cases a portion of the rounded end of the pillar formed the base of a cone, whose vertex was in or near the axis of the pillar. This cone acting as a wedge whose sides were in the angle of least resistance, and having its vertex sharp, split and cut up the sides of the pillar, of which it formed a part*.

All the pillars in the preceding abstract became bent, and broke in the middle, as if the breaking weight had been transverse. This was the case with all the pillars tried, till those of ·52 inch diameter became only two inches long, the length being less than four times the diameter.

Some of the short pillars, it will be noticed, bore more than the average crushing weight, arising from these having been cast very small, and therefore being harder and resisting with more than the average strength of the metal.

* For more detailed information on the crushing of short masses of cast iron, and the interesting forms of its fractures, see the Report in the volume of the British Association before referred to.

8. We see, therefore, that when pillars of cast iron, with flat ends, are shorter than about thirty times the diameter, the ratio of the strength of those of the same dimensions, with flat and with rounded ends, is very variable. But when the length is thirty times the diameter, or more, up to one hundred and twenty-one times, we have shown it to be nearly constant; and we may infer that the ratio would be constant, however greatly the length might exceed the diameter; since the uniformity seems to depend upon the circumstance that the weight necessary to break these long pillars by flexure, is not sufficiently great to produce much compression in them, and therefore does not alter sensibly the position of their neutral line. In this respect, cast iron has the advantage of most other metals, on account of its great power of resisting crushing, which is, on the average, from six to seven times as much as that necessary to tear it asunder*. Wrought iron, on the other hand, does not require nearly so much force to crush it, so as in a great measure to destroy its utility†, as is required to tear it asunder (Art. 60.); and therefore the ratio of the strength of equal pillars, with rounded and flat ends, becomes variable somewhat sooner in it than in cast iron, though the comparative results are the same in long pillars. The experiments on timber, and on steel, so far as they have gone, give the same results with respect to long pillars.

9. Confining our researches, for the present, exclusively to what may be termed *long cast-iron pillars*, or those whose length, in pillars with flat ends, is thirty times the diameter, or upwards, since these are but little compressed by the breaking weight, we shall obtain some interesting properties. These properties, doubtless, would be common to all rigid bodies, where the length, compared with the diameter, was so great that the breaking weight was not sufficient to diminish sensibly the power of the body to resist a transverse force. Till recently, in all inquiries respecting the strength of materials, bodies have been assumed to be incompressible; and if they really were so, the constant ratio above-mentioned, and some other properties, which I shall deduce from the results of the experiments upon cast-iron pillars, would be found in them, whatever might be their length, compared with their diameter. Some descriptions of stone may perhaps approach much nearer than cast iron to this state of ultimate incompressibility, compared with their power of resisting tension.

Strength of Long Pillars of Cast Iron.

10. We have seen, that in all long pillars, where the dimensions are the same, the resistance to crushing by flexure is about three times as great when the ends are flat as when they are rounded. This general agreement, and the near approach in point of strength, in pillars with flat ends, to those of the same diameter and of half the length with rounded ends (as may be seen by comparing the preceding abstract), led me to conceive, that if the pillars with flat ends were more firmly fixed at the extre-

* Transactions of the British Association, vol. vi., before referred to.

† The valuable experiments of Mr. Barlow refer to a further destruction of the material.

mities, their strength would be increased, so that they would bear the same as pillars of equal diameter and of half the length with the ends rounded.

To ascertain whether this would be the case, experiments were made upon pillars 60½ inches long, from the same models as were made in Tables I. and II., but with this addition, the pillars were now cast with strong discs upon their ends, of about double the diameter of the pillar. These discs were turned flat and perpendicular to the axis, and therefore rendered the ends of the pillar, when placed between the parallel crushing surfaces, perfectly immoveable. The results of these experiments are given in Table IV.

11. Comparing the mean breaking weights in Table IV. with those of the corresponding pillars in Table II., we shall find that pillars with discs on the ends are somewhat stronger than those without them. I will select the mean results from the second and fourth Tables, from models of the same diameter and length; and from the first Table, those from pillars of the same diameter and half the length; affixing to each result a number in a parenthesis, indicating the number of experiments the mean has been derived from.

Abstract from Tables I., II., and IV.

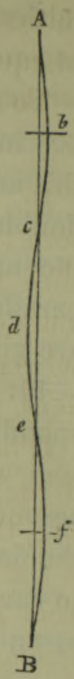
Pillars with their ends turned flat. Length 60½ inches. (Table II.)		Pillars with discs on the ends turned flat. Length 60½ inches. (Table IV.)		Pillars with the ends rounded. Length 30¼ inches. (Table I.)	
Diameter. inch.	Breaking weight. lbs.	Diameter. inch.	Breaking weight. lbs.	Diameter. inch.	Breaking weight. lbs.
		·51	487 (2)	·50	539 (3)
·77	2456 (2)	·775	2719 (2)	·77	2726 (1)
·997	6238 (3)	1·00	6830 (4)	·99	6105 (2)
1·29	16064 (3)	1·28	16369 (2)	1·29	17235 (2)
1.56	28962 (3)	1·53	30789 (2)	1·52	32531 (3)

12. These results are, as we see, the means from a considerable number of experiments, agreeing moderately well among themselves: they show, as has been observed before, that the discs give the pillar a small increase of strength above that of a pillar with flat ends only; and the approach to equality between the strengths of the pillars with discs, and of those of the same diameter, and half the length with rounded ends, is perhaps as near as can be expected in experiments of this nature. Hence we may conclude, *that a long uniform cast-iron pillar, with its ends firmly fixed (whether by means of discs or otherwise), has the same power to resist breaking, as a pillar of the same diameter and half the length with the ends rounded, or turned so that the force would pass through the axis.*

It may here be observed, that, in comparing the strengths, I have, as before, neglected the slight differences of diameter in pillars from the same model, and supposed them all of equal diameter.

13. Of the general conclusion arrived at above, I would offer the following as an explanation. Suppose a long uniform bar, or bolt of cast iron, were bent by a press-

ure at its ends, so as to take the form A *b c d e f* B; where all the curves A *b c*, *c d e*, *e f* B, separated by the straight line A *c e* B, would be equal, since the bar was supposed to be uniform. The curve having taken this form, suppose the points *b* and *f* to be rendered immoveable by some firm fixings at those points. This done, it is evident we may remove the parts near to A and B, without at all altering the curve *b c d e f* of the part of the pillar between *b* and *f*, and consider only that part. The part *b f*, which alone we shall have to consider, will be equally bent at all the points *b*, *d*, *f*. The parts *c* and *e*, too, are points of contrary flexure; consequently the pillar is not bent in them. These points are unconstrained, except by the pressure which forces them together; and the pillar might be reduced to any degree in them, provided they were not crushed or detruded by the compressing force. These points may then be conceived as acting like the rounded ends in the pillars of Table I.; and the part *c d e* of the pillar, with its ends *c* and *e* supposed to be rounded, will be bearing the same weight as the whole pillar *b c d e f*, of double the length, with its ends *b f* firmly fixed.



14. The theory of the strength of pillars, as given by Euler and Lagrange*, and afterwards pursued by Poisson† and others, furnishes us with little information upon these subjects. According to that theory, the strength is inversely as the square of the length; or a pillar, half the length of another of the same diameter, would have four times the strength. The results of my experiments give the strength nearer to three times, as a general value for the half length in cast iron. It is, however, very variable, as will be seen by the first and second tables, unless we restrict ourselves to the longer columns.

The strength is much influenced, as has been previously observed, by the quantity of compression which the pillar sustains; and, consequently, by the position of the neutral line when the pillar is bent. The strengths, too, are different in their definition in the two cases. In the theory of Euler, the strength is estimated by the greatest weight which a pillar would bear without flexure; whilst in the present case, the estimate is formed upon the weight which would break the pillar by flexure. I have sought, on many occasions, but without success, to determine experimentally some fixed point, according to the definition of the continental theory. So far as I can see, flexure usually commences with very small weights, such as could be of little use to load pillars with in practice. It seems to be produced by weights much smaller than are sufficient to render it capable of being measured. I am, therefore, doubtful whether such a fixed point will ever be obtained, if indeed it exists. With respect to the conclusions of some writers, that flexure does not take place with less than about half the breaking weight, this, as is evident from my experiments, taken in general, could only mean large and palpable flexure; and it is not improbable that the writers were in some degree deceived from their having generally used specimens

* Acad. de Berlin, 1769; Collection Académique de Turin, Vol. 5. † Poisson, Mécanique, 2nd edit.

thicker, compared with their lengths, than have usually been employed in this paper. Some results of the continental theory we shall, however, find of great service further on.

Pillars with one End rounded and the other flat.

15. In the first and second Tables we have the strength of cast-iron pillars, with both their ends rounded, and both flat; we will now refer to Table V., which contains the results of experiments to ascertain the strength of pillars with one end rounded and the other flat, and in some cases with discs; these results being placed between those from the pillars, with round and flat ends, as taken, with some additions, from the first and second Tables.

It has been shown, that the strength of pillars, with both their ends fixed, is to the strength, when the ends are rounded, in a constant ratio. It seems probable, therefore, that the strength, when one end is fixed and the other rounded, must be related to the results of the other cases by some very simple law. Taking the mean results from Table V. and other places, and neglecting the slight differences in diameter among castings from the same model, we have as below:

	Breaking weights in lbs.				
Both ends rounded	143	3017	7009	7009	16493
One end rounded and one flat, or with a disc ..	256*	6278	13499	13565	33557
Both ends flat, or with discs................	487	9007	20310	22475	

16. The pillars in each vertical column in this abstract, are of the same length and diameter; the strengths, therefore, in the three different cases, reading downwards, are as 1, 2, 3, nearly, the middle being an arithmetical mean between the other two. It is shown, moreover, by the experiments further on, upon timber, wrought iron, and steel, that *the strength of a pillar with one end round and the other flat, is always an arithmetical mean between the strengths of pillars of the same dimensions with both ends rounded and both flat,* however the ratio of the strength of these may vary†. We see, too, by the first and second columns in the preceding abstract from Table IV., which contain the ratio of the strengths in pillars with discs, that their results are in accordance with the rest. From these facts, as well as from other evidence not adduced, I conclude that the above mean would always take place, however the fixed ends were fastened.

17. The case of pillars fixed at one end and moveable at the other, is not without its interest in practice; as piston rods of steam-engines and the rods of force-pumps are instances of the use of materials in that form, it being the same thing in effect whether the pillar be rounded at the end, or turn upon a joint there. Other instances may often be seen in practice; as where pillars are firmly based at the bottom, but

* This pillar was weakened a little by a previous experiment upon it.

† The experiments on wrought iron, show that it is, practically, the case even when the pillars are much crushed by the pressure.

through imperfect fixings, or some other cause, are capable of moving at the top. It applies, too, if I am not mistaken, to the experiments upon timber by GIRARD *, whose pillars rested upon a flat surface at the bottom, but turned with a joint at the top, or were moveable there in the direction of the length of the lever. But when the pillars were so placed as to have their least dimensions in that direction, they broke with the mean weight, as above defined, instead of the full weight, which they were expected to bear, as will be evident from a careful inspection of M. GIRARD's plates attached to the work.

18. In all the previous experiments, where the pillars were uniform and the same at both ends, the fracture took place in the middle, or very near to it. This, as might be expected, was not the case where one end was flat and the other rounded; in that case the pillar broke at some point nearer to the rounded end, the piece broken off being always a little more than one-third of the whole length. The excess, however, above one-third was so small, that I conceive this division would be absolutely that to which the material would tend, provided it were incompressible, and if the pillar were better fixed at the flat end, and more free to move at the rounded one.

With this view, two pillars, each thirty inches and a quarter long, and one inch in diameter, were formed with broad discs on one end, and rendered as nearly pointed at the other as was consistent with their bearing the pressure.

The result of the experiment upon these was this: the part broken off from one was 10·42 inches, and from the other 10·30, whilst the one-third part of the whole length was 10·08 inches. I was led to conceive the division above (one-third) not improbable, since it had been shown before, that a pillar with flat ends, or with discs upon them, bore the same as a pillar of half the length with rounded ends; and as the pillars above had a disc on one end, and were rounded at the other, they might divide themselves so that the part with the flat end might be twice as long as that with the rounded one.

The results of my experiments on wrought iron, which is the only material besides cast iron in which I have examined the matter, give the division so as to make the piece, near to the rounded end, more in excess above one-third of the whole length than in cast iron: the experiments, however, were made upon pillars too short to try the matter quite satisfactorily. This deviation, if the above views are correct, may perhaps be attributed to the greater compressibility of wrought iron than of cast, as compared with their powers of resisting tension. A better material to try it would be stone.

19. We have now seen, that when uniform pillars are so placed that one end is fixed and the other moveable, the point of greatest strain is at one-third, or a little more, of the whole length of the pillar from the moveable end. We might, therefore, economise the metal by increasing the thickness in that point, and reducing the strength as we recede both ways from it.

* Traité Analytique de la Résistance des Solides.

Pillars enlarged in the Middle.

20. It has been stated above, that long pillars of uniform thickness, with both ends alike, always break first, in or near to the middle: this was the case even when they had discs upon the ends, to give the utmost firmness to their fixings. It seemed evident, then, that the pillar was always too weak there. I felt, therefore, desirous to ascertain the effect of strengthening the middle of pillars; but as the ends could not be reduced at pleasure, since they would be crushed without bending, in the manner of our shortest pillars in Table II., I increased the diameter in the middle, leaving that of the ends the same; and conceiving it best to make the experiments upon pillars whose form was as simple as possible, models were made of the form of two frustums of cones, the bases of which met in the middle of the pillar, the end diameter being 1 inch, and the middle $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$, 2 inches. The sides, therefore, were straight, and regularly tapering from the middle to the two ends. Tables VI. and VII. contain the results of the experiments, the pillars being all of the same length. In the first of these Tables, the extreme ends of the pillars were rounded, that the pressure might be through the axis; and in the second, the ends had large discs upon them, turned flat.

Table VI. shows, that in all the pillars with rounded ends, those with increased middles were stronger than uniform pillars of the same weight, the increase being about one-seventh of the weight borne by the former.

In the pillars with discs, Table VII., those with the middle but little increased, had no advantage, with regard to strength, over the uniform ones. But the pillars with the middle diameter half as great again as the end ones, bore from one-eighth to one-ninth more than uniform pillars of the same weight with discs upon the ends.

Strength of long Pillars as dependent upon their Dimensions.

21. We shall now investigate the relative strengths of long pillars, as influenced by variations in their diameter and length. From the theory of Euler, it would appear that the power of a pillar to resist *incipient flexure* is directly as the fourth power of the diameter, and inversely as the square of the length. The inquiry in this paper is with respect to the resistance of pillars to *fracture*; and, as has been mentioned before, I have not been able to find the point to which Euler's computations refer. His measures of the strength, however, seem not very widely different from those which apply to the breaking point.

Strength as dependent upon the Diameter.

22. I shall first ascertain, from the first and second Tables, the power n of the diameter to which the strength is proportional, in pillars of the same length. Comparing the resistances of the pillars whose diameters were ·50 and 1·765 inches, and the length $60\frac{1}{2}$ inches, in Table I., we find that the former was broken with 143 lbs., and the latter with 15560.

Since $\frac{1{\cdot}765}{{\cdot}50} = \frac{3{\cdot}530}{1}$

we have $1^n : 3{\cdot}53^n :: 143 : 15560,$

and $3{\cdot}53^n = \frac{15560}{143};$

whence

$$n = \frac{\text{Log.} \frac{15560}{143}}{\text{Log. } 3{\cdot}53} = 3{\cdot}718.$$

I insert below values of n, thus obtained, from the first and second Tables; and from the square pillars and those with discs, in Tables III. and IV., beginning with long pillars with their ends rounded.

23. In Table I., comparing the strength of a pillar of

·50 in. diameter, and $60\frac{1}{2}$ ins. long, with that of ·77 in. diameter, gives $n = 3{\cdot}928$
·50 in. diameter, and $60\frac{1}{2}$ ins. long, with that of ·99 in. diameter, gives $n = 3{\cdot}788$
·50 in. diameter, and $60\frac{1}{2}$ ins. long, with that of 1·29 in. diameter, gives $n = 3{\cdot}889$
·50 in. diameter, and $60\frac{1}{2}$ ins. long, with that of 1·52 in. diameter, gives $n = 3{\cdot}894$
·50 in. diameter, and $60\frac{1}{2}$ ins. long, with that of 1·765 in. diameter, gives $n = 3{\cdot}718$
·50 in. diameter, and $60\frac{1}{2}$ ins. long, with that of 1·94 in. diameter, gives $n = 3{\cdot}741$
·77 in. diameter, and $60\frac{1}{2}$ ins. long, with that of 1·765 in. diameter, gives $n = 3{\cdot}609$
·77 in. diameter, and $60\frac{1}{2}$ ins. long, with that of 1·94 in. diameter, gives $n = 3{\cdot}654$
·50 in. diameter, and $30\frac{1}{4}$ ins. long, with that of 1·29 in. diameter, gives $n = 3{\cdot}656$
·50 in. diameter, and $30\frac{1}{4}$ ins. long, with that of 1·52 in. diameter, gives $n = 3{\cdot}687$
·77 in. diameter, and $30\frac{1}{4}$ ins. long, with that of 1·52 in. diameter, gives $n = 3{\cdot}637$
·99 in. diameter, and $30\frac{1}{4}$ ins. long, with that of 1·52 in. diameter, gives $n = 3{\cdot}902$
·50 in. diameter, and $15\frac{1}{8}$ ins. long, with that of ·76 in. diameter, gives $n = 3{\cdot}768$
·50 in. diameter, and $15\frac{1}{8}$ ins. long, with that of ·99 in. diameter, gives $n = 3{\cdot}425$

Mean value of $n = 3{\cdot}736$

In this comparison I have excluded all pillars, the length of which was not greater than about fifteen times the diameter. In pillars shorter than in about this proportion, a great change doubtless takes place by degrees in their elasticity, and their strength is much reduced. This remark is intended to be understood only of pillars with rounded ends: in those with flat ends, the same change, we have seen, begins to take place when the length of the pillar is less than thirty times the diameter. As this change in the elasticity of pillars must be produced by the same weight (the diameter being supposed constant), the relative strength of pillars with rounded and with flat ends, to resist the change, must depend upon the *laws* that regulate their ultimate strength; and we have seen that a pillar with rounded ends will bear the same as one of double the length with flat ones. In the experiments, Table III., on square bars

$60\frac{1}{2}$ inches long, rounded at the ends, comparing together the strengths of the pillars whose sides are

$$\cdot7716 \text{ and } 1\cdot0275, \text{ gives } n = 3\cdot9830;$$
$$\cdot7716 \text{ and } 1\cdot540, \text{ gives } n = 3\cdot6789;$$
$$1\cdot0275 \text{ and } 1\cdot540, \text{ gives } n = 3\cdot4698;$$
$$\text{Mean } n = 3\cdot7105.$$

24. From the two preceding inquiries, as to the value of n, we see, that whether the pillar be circular or square, the power of the thickness to which the strength is proportional is nearly the same. We may therefore conclude, that the same value of n would be obtained from any other form of section.

In the values obtained from the circular pillars, which include specimens varying in length from twenty times the diameter to one hundred and twenty times, the values of n do not differ so widely, but that they may be considered as constant. They in no case rise so high as four, the power according to EULER; but in pillars, so long with respect to their diameter that the breaking weight is very small, they approach nearly to it. We may, therefore, say that four is the power to which they would arrive if the breaking weight was so small that it produced no compression in the pillar.

Long Pillars with the Ends flat.

25. We will determine the values of n, the power of the diameter to which the strength is proportional, in these pillars, in the same manner as was done in the former.

In the cylinders $60\frac{1}{2}$ inches long (Table II.), comparing the strength of that of

$$\cdot77 \text{ inch diameter with that of } \cdot997, \text{ gives } n = 3\cdot606;$$
$$\cdot77 \text{ inch diameter with that of } 1\cdot29, \text{ gives } n = 3\cdot641;$$
$$\cdot77 \text{ inch diameter with that of } 1\cdot56, \text{ gives } n = 3\cdot495;$$
$$\cdot997 \text{ inch diameter with that of } 1\cdot29, \text{ gives } n = 3\cdot670;$$
$$\cdot997 \text{ inch diameter with that of } 1\cdot56, \text{ gives } n = 3\cdot428;$$
$$\text{Mean } n = 3\cdot568^*.$$

The pillars of the same length, with discs upon the ends, in Table IV., if we make the same comparison of strength, give as below:

Comparing that of

$$\cdot51 \text{ inch diameter with that of } 1\cdot00 \text{ diameter, we have } n = 3\cdot922;$$
$$\cdot51 \text{ inch diameter with that of } 1\cdot28 \text{ diameter, we have } n = 3\cdot820;$$
$$\cdot51 \text{ inch diameter with that of } 1\cdot53 \text{ diameter, we have } n = 3\cdot775;$$
$$\cdot775 \text{ inch diameter with that of } 1\cdot53 \text{ diameter, we have } n = 3\cdot568;$$
$$\cdot775 \text{ inch diameter with that of } 1\cdot28 \text{ diameter, we have } n = 3\cdot578;$$
$$1\cdot00 \text{ inch diameter with that of } 1\cdot53 \text{ diameter, we have } n = 3\cdot412;$$
$$\text{Mean} = 3\cdot679.$$

* In my earlier experiments the mean obtained was 3·55, and that number will be used in the computations further on in this paper.

The experiments in Table II., upon pillars shorter than these, are not in sufficient variety to enable us satisfactorily to pursue the comparison; and if we attempted one between the strengths of the short pillars in that Table, as, for instance, those of one-third, or one-fourth the length of those above, we should find the value of n very low, on account of the crippling or other great change which we have seen (Art. 6) that pillars sustain by the weight, when they are made shorter than about thirty times the diameter.

Strength as dependent on Length.

26. In order to obtain the power of the length, to which the strength of pillars of the same diameter is proportional, considering this as some inverse power x, we will determine its value from the first and second Tables, beginning with Table I., in which the lengths vary as widely as eight and sixteen to one; and we shall take the values from the whole range in that Table.

27. Here we find that a pillar of half an inch diameter and 60·5 inches long, was broken with 143 lbs., and one of the same diameter and 3·78125 inches, the one-sixteenth of the former length, required 15107 lbs. to break it. Proceeding in the manner adopted before, we have,

$$1^x \; : \; 16^x \; :: \; 143 \; : \; 15107.$$

Whence we obtain $x = 1{\cdot}6807$.

In this comparison I shall assume all the pillars which were cast from the same models, to be equally thick, neglecting occasional slight differences in their diameters.

28. *Pillars with their Ends rounded.*

Diameters of cylinders considered equal.		Lengths of which the strengths are compared.		Values of x.	Mean values of x.
inch.	inch.	inches.	inches.		
·50	·50	60·5	and 30·25	1·914	
·50	·50	60·5	15·125	1·867	1·799
·50	·497	60·5	7·5625	1·734	
·50	·50	60·5	3·78125	1·681	
·77	·77	60·5	30·25	1·805	
·77	·76	60·5	15·125	1·782	
·77	·77	60·5	7·5625	1·626	1·702
·77	·76	30·25	15·125	1·758	
·77	·77	30·25	7·5625	1·537	
·99	·99	60·5	30·25	1·682	1·685
·99	·99	60·5	15·125	1·688	
1·29	1·29	60·5	30·25	1·595	1·595
1·52	1·52	60·5	30·25	1·583	1·583

29. *Pillars with their Ends flat.*

Diameters considered equal.		Lengths of which the strengths are compared.		Values of x.	Mean value of x.
inch.	inch.	inches.	inches.		
·77	·77	60·5 and	30·25	1·843	1·625
·77	·777	60·5	20·1666	1·682	
·77	·775	60·5	15·125	1·550	
·77	·785	60·5	12·100	1·424	
·997	1·01	60·5	30·25	1·691	1·587
·997	1·022	60·5	20·1666	1·483	

30. The value of x, as exhibited in the preceding abstracts from Tables I. and II., is not constant. Taking pillars of any particular diameter, we see that x decreases according as we decrease the length of the pillar; and consequently increase the pressure necessary to break it. Thus, in pillars with rounded ends, taking those of ·50 inch diameter, and comparing the strengths of those of $60\frac{1}{2}$ inches long with those of 30·25, 15·125, 7·5625, 3·78125 inches, we have $x=1·914$, 1·867, 1·734, 1·681. In pillars with flat ends, taking those of which the diameter is ·77 inch nearly, and comparing those of $60\frac{1}{2}$ inches long with those of 30·25, 20·1666, 15·125, 12·1 inches, we obtain successively, $x=1·843$, 1·682, 1·550, 1·424. This decrease in the value of x also takes place in the comparison between the different lengths of the pillars ·99 inch diameter; and it is evident that it would do so in all cases where the diameter was the same, whatever that diameter might be. The falling off in the strength of pillars, whose length is less than twenty-five to thirty times the diameter, in those with flat ends; and about one-half that length, in those with rounded ends, as before observed, is not the sole cause of this diminution in the value of x, though it is a part of it.

Indeed, we may perhaps say that both depend in some degree upon the same cause, the compressibility of the material. The increasing weight alters the position of the neutral line in the pillar, and consequently lessens the part subjected to tension. By this means the flexibility of the pillar is increased, and its resistance decreased; because this resistance depends upon the transverse strength and the power to resist flexure. The position of the neutral line, on which this variation of the strength depends, has been obtained in several of the cases of Tables I. and II.; and indeed many others.

31. In the abstract from Table I., just given, and in which the lengths vary as widely as sixteen to one, we find the highest value of $x=1·914$, and the lowest $=1·537$; and the mean from all the values $=1·7117$.

The abstract from the pillars with flat ends, in Table II., is not pursued to the same extent as the former. In it, the highest and lowest values of x are 1·843 and 1·424; the mean from the whole of these being 1·612.

32. The pillars compared above comprise a much greater range of lengths than those in general use; and as the shortest pillars do not come within the simple rules applicable to the longer ones, I shall adopt 1·7 in the future calculations as an approximate value of x, applicable to most of the pillars used in practice.

33. We see from the preceding abstracts, that the values of x never rise so high as two, but approach toward it accordingly as the breaking weight becomes smaller. In the pillars ·77 inch diameter and $60\frac{1}{2}$ inches long, where the length is seventy-eight times the diameter, the value of x is 1·805 in one case, and 1·843 in another; and in the pillars ·50 inch diameter, and $60\frac{1}{2}$ inches long, the length being one hundred and twenty-one times the diameter, the value of x is 1·914. These facts, and the regular increase in the value of x according as the breaking weight is diminished, show that 2 is the value to which x would approximate if the breaking weight were infinitely small, or the body perfectly incompressible.

Computations of the Strength of Long Pillars by means of the preceding Numerical Results.

34. We have found that the strength of cast-iron pillars, whose diameter is the same, is inversely as the 1·7th power of the length nearly; and where the diameter differs, the strength varies according to a power of the diameter which is nearly constant. This, in cylindrical pillars, whose ends are rounded, we have found to be 3·736, and in those with flat ends 3·568, the pillars with discs upon the ends giving 3·679.

The earlier experiments gave 3·76 for the pillars rounded at the ends, and 3·55 for the flat ones; and these numbers will be retained in the following computations, whatever the form of the section may be; it having been shown that the power was nearly the same, whether the pillars were round or square.

35. As a convenient unit of comparison, we may determine the strength of a solid pillar whose length is one *foot* and diameter one *inch*, as obtained by calculation from most of the experiments in the first and second Tables.

Putting then d for the diameter of a given pillar in inches, l for its length in feet, w for its breaking weight, and x for that of a pillar one inch diameter and one foot long, we have $\frac{d^{3\cdot76}}{l^{1\cdot7}}$ as a comparative measure of the strength in pillars with rounded ends. Whence

$$\frac{d^{3\cdot76}}{l^{1\cdot7}} : \frac{1^{3\cdot76}}{1^{1\cdot7}} :: w : x$$

$$\therefore x = \frac{w\,l^{1\cdot7}}{d^{3\cdot76}},$$

for pillars with their ends rounded.

In the same manner, for pillars with their ends flat,

$$x = \frac{w\,l^{1\cdot7}}{d^{3\cdot55}}.$$

The results from the pillars with *rounded* ends in Table I. are as follow:

Diameter of pillar.	Length.		Breaking weight.	Value of x.	Mean value of x.	Error from using the mean, in parts of the value of x.
inch.	inches.	feet.	lbs.	lbs.		
·50	60·5	= 5·04166	143	30309	33379 lbs. = 14·901 tons.	$+\frac{1}{9\cdot8}$
·50	30·25	= 2·5208	539	35162		$-\frac{1}{19\cdot7}$
·50	15·125	= 1·2604	1904	38230		$-\frac{1}{7\cdot8}$
·497	7·5625	= ·6302	5262	33263		$+\frac{1}{28\cdot9}$
·77	60·5	= 5·0416	780	32603		$+\frac{1}{42}$
·77	30·25	= 2·5208	2726	35070		$-\frac{1}{20\cdot7}$
·76	15·125	= 1·2604	9223	38360		$-\frac{1}{7\cdot6}$
·76	10·083	= ·84028	17506	36547		$-\frac{1}{11\cdot5}$
·99	60·5	= 5·04166	1902	30902		$+\frac{1}{12\cdot4}$
·99	30·25	= 2·5208	6105	30529		$+\frac{1}{10\cdot7}$
1·00	20·1666	= 1·6805	15737	36639		$-\frac{1}{11\cdot2}$
·99	15·125	= 1·2604	19752	30401		$+\frac{1}{10\cdot2}$
1·29	60·5	= 5·04166	5707	34273		$-\frac{1}{38\cdot2}$
1·29	30·25	= 2·5208	17235	31857		$+\frac{1}{20\cdot9}$
1·5275	60·5	= 5·0416	10750	34229		$-\frac{1}{40\cdot2}$
1·52	30·25	= 2·5208	32531	32447		$+\frac{1}{34\cdot8}$
1·772	60·5	= 5·0416	16562	30092		$+\frac{1}{9\cdot1}$
1·95	60·5	= 5·0416	23551	29901		$+\frac{1}{8\cdot6}$

The enumeration above includes pillars varying in length from 121 times the diameter down to 15 times, or as 8 to 1. Through this range the greatest error arising from using the mean value of x here obtained is $\frac{1}{7\cdot6}$, or somewhat more than one-eighth. We shall, therefore, take 33379 lbs., or 14·9 tons, as a unit of measure for the strength of a solid uniform cast-iron pillar, of Low Moor iron, No. 3, one foot long and one inch diameter, the ends of which are rounded.

36. For solid pillars of other dimensions with rounded ends, in the same material, taking the length in feet and the diameter in inches, and using the same notation as before, we have w the strength as below:

$$w = 33379\,\frac{d^{3\cdot76}}{l^{1\cdot7}} \text{ for the strength in lbs.}$$

$$w = 14\cdot9\,\frac{d^{3\cdot76}}{l^{1\cdot7}} \text{ for the strength in tons.}$$

This rule is only intended to apply to pillars whose length is fifteen times the diameter or upwards, and in the largest pillars, perhaps, not so low. It appears probable that we should find a falling off in the strength of these when they are considerably more than fifteen times the diameter in length.

This reduction in the resistance of the thicker pillars, compared with what we are led to expect from the results of the smaller, arises, as appears to me, from the greater softness of the metal when cast in large masses than in small; and this cause, I conceive, has some influence upon the strength of all cast-iron pillars.

37. To find x, the strength of a pillar one inch diameter and one foot long, in solid cylinders flat at the ends, from the mean results in Table II.

Using here the formula

$$x = \frac{w\,l^{1 \cdot 7}}{d^{3 \cdot 55}},$$

and restricting ourselves to cylinders whose length is upwards of twenty-five times the diameter, we have,

Diameter of pillar.	Length of pillar.		Breaking weight.	Value of x.	Mean value of x.	Error from using the mean, in parts of the value of x.
inch.	inches.	feet.	lbs.	lbs.		
·51	60·5	= 5·04166	487	83182*	98922 lbs. = 44·16 tons.	
·77	60·5	= 5·04166	2456	97175		
·50	30·25	= 2·5208	1662	93735		
·77	30·25	= 2·5208	8811	107300		
·777	20·166	= 1·6805	15581	92229		$+\frac{1}{13 \cdot 78}$
·997	60·5	= 5·0416	6238	98638		
1·01	30·25	= 2·5208	20310	94403		
1·29	60·5	= 5·0416	16064	101770		
1·56	60·5	= 5·0416	28962	93455		
·51	20·166	= 1·68055	3830	101063		
·51	15·125	= 1·2604	6764	109445		$-\frac{1}{9 \cdot 4}$

The abstract above contains pillars varying in length from 25 to 121 times the diameter, and omitting the first, for the reason given in the note, the greatest error which has occurred by using the general mean 98922 lbs., is $\frac{1}{9 \cdot 4}$th. Hence we may adopt 98922 lbs., or 44·16 tons, as the strength of a cast-iron pillar, one inch diameter and one foot long, flat at the ends; this number, however, only to be applied to pillars whose length is 30 times the diameter, or upwards.

38. Retaining the same notation as before, we have, for solid cylindrical pillars of any given size, the lengths being as above:

$$w = 98922 \times \frac{d^{3 \cdot 55}}{l^{1 \cdot 7}}, \text{ for the strength in lbs.}$$

$$w = 44 \cdot 16 \times \frac{d^{3 \cdot 55}}{l^{1 \cdot 7}}, \text{ for the strength in tons.}$$

* I conceive this result to be too low, on account of the difficulty of having slender pillars quite straight. It is, therefore, not included in the general mean value of x.

Short Pillars.

39. The mean value 98922 lbs. would be too great, if used for calculating, by the preceding formula, the strengths of solid round pillars, which are shorter than about 30 times the diameter. I shall, therefore, insert below the values of x, from the shorter pillars in Table II.; together with the particulars of the experiments from which they were derived.

Putting these in a line with the real breaking weights, and the number of tons per square inch, upon the section of the pillars, I will endeavour to show the cause of the falling off in strength.

No. of experiment.	Diameter.	Length of pillar.			Breaking weight.	Tons per square inch, the crushing weight being 49 nearly.	Value of x from formula $\frac{w\,l^{1\cdot7}}{d^{3\cdot55}}$	Ratio of diameter to length.
	inch.	inches.		feet.	lbs.		lbs.	
1	·777	20·166	=	1·6805	15581	14·7	92224	1 : 25·95
2	.50	12·1	=	1·0083	7195	16·4	85467	1 : 24·20
3	·50	10·083	=	·84028	8931	20·3	77817	1 : 20·166
4	1·022	20·166	=	1·6805	31804	17·3	71154	1 : 19·732
5	·775	15·125	=	1·2604	21059	19·9	77141	1 : 19·516
6	·785	12·1	=	1·0083	24287	22·4	46207	1 : 15·414
7	·50	7·5625	=	·6302	11255	25·6	60133	1 : 15·125
8	1·00	15·125	=	1·2604	40250	22·9	59516	1 : 15·125
9	·768	10·083	=	·84028	25923	25·0	49224	1 : 13·129
10	·777	7·5625	=	·6302	32007	30·1	35757	1 : 9·733
11	·50	3·7812	=	·3151	17468	39·7	28725	1 : 7·562
12	·52	2	=	·16666	22867	48·1	11080	1 : 3·846
13	·52	1	=	·08333	24616	51·74		1 : 1·923

40. The second, third, and fourth columns in this abstract give, as will be seen, the dimensions and breaking weight of the pillars, each result being generally the mean from several experiments. The fifth column gives the number of tons which the breaking weight is upon the area of the section; the sixth, the value of x, or the strength of a pillar, one foot long and one inch diameter, as calculated from each result by the formula before used (Art. 38.). The last column gives the number of times which the length exceeds the diameter.

If there were no falling off in the strength of these pillars through crushing, as before mentioned (Art. 6. 7.), we know that the value of x would be constant, and nearly equal to the mean value, 98922 lbs.; which applies, as we have seen, to cast-iron pillars flat at the ends, of which the length is not less than about 30 times the diameter. But we see, from the abstract above, that in pillars whose length is about 15 times the diameter, the value of x, which being obtained from the breaking weight must fall with it, is reduced to 46207, 60133, 59516, mean 55285; being somewhat more than 49461, the half of the general mean. In pillars whose length is 7·56 times the diameter, or a little more than a quarter of 30 times the diameter, the strength is reduced to 28725, whilst the fourth of the general mean is 24731. Pillars whose length is about 20 times the diameter, or two-thirds of 30 times, we see give the value of x 77817, 71154, and 77141, whilst two-thirds of the general mean is 65948 lbs.

All those values of x are in excess of the part of the general mean they are compared with. We might, therefore, obtain a rough approximation to the strength of short pillars by calculating, by the formula before used, the strength of the pillar; and then taking such a portion of this calculated strength as the length of the pillar is of what it would have been if it were 30 times the diameter. This easy rule would nearly in all cases give the strength somewhat under the real strength, as appears from the instances we have cited, and others we might take from the abstract.

More correct mode of estimating the strength.

41. We have seen, that when pillars are reduced in length beyond a certain extent, there is a reduction in their strength from that which is given by the rules for longer pillars; and this falling off is nearly in proportion to the reduction in the length of the pillar. Another mode of considering the matter will, it is conceived, throw some light upon the reason of the conclusion in the last article, and furnish better means of estimating the strengths.

The two last pillars, in the abstract last given (Art. 39.), broke by being crushed, all the others having been broken by flexure. One broke with 48·1 tons per square inch; and the other with 51·7. The mean result of the experiments, upon the resistance to crushing in this metal, gave 49 tons per square inch nearly (Art. 55.). Taking a mean between the results of such pillars in the abstract as have their length the same number of times the diameter nearly, we have as below, in experiments 3, 4 and 5, a pressure of 19·16 tons answering to a value of $x = 75370$ lbs.; a mean from the experiments 6, 7, 8 and 9, gives a pressure of 23·97 tons per inch, answering to a value of $x = 53770$ lbs.; and a mean between the results in the 10th and 11th experiments, gives 34·9 for the pressure per inch, and the value of $x = 32241$.

We therefore see that $\frac{19{\cdot}16}{49}$ of the crushing weight reduced the value of x from 98922 to 75370; $\frac{23{\cdot}97}{49}$ reduced it to 53770; and $\frac{34{\cdot}9}{49}$ reduced it to 32241. In other words, when the pressure is a little more than two-thirds of the crushing weight, the strength to bear flexure is reduced to less than one-third; when the pressure is a little under half of the crushing weight, the value of x is somewhat more than half its first value; and when the pressure is less, the reduction is diminished. Comparing the weights, per square inch, with all the values of x, in the two columns, we find, amidst many anomalies, that the weights increase in some such proportion as the values of x decrease; and the latter become very small where the pressure approaches to the crushing force. Considering then the pillar as having two functions, one to support the weight and the other to resist flexure, it follows, that when the material is incompressible (supposing such to exist), or when the pressure necessary to break the pillar is very small, on account of the greatness of its length compared with its lateral dimensions, then the strength of the whole transverse section of the pillar will be employed in resisting flexure; when the breaking pressure is one-half of what would

be required to crush the material, one-half only of the strength may be considered as available for resistance to flexure, whilst the other half is employed to resist crushing; and when, through the shortness of the pillar, the breaking pressure is so great as to be nearly equal to the crushing force, we may consider that no part of the strength of the pillar is applied to resist flexure.

We may, therefore, separate these effects by taking, in imagination, from the pillar (by reducing its breadth, since the strength is as the breadth) as much as would support the pressure, and consider the remainder as resisting flexure to the degrees indicated by the previous rules (Arts. 36 and 38.).

42. Suppose then c to be the force which would crush the pillar without flexure; d the utmost pressure the pillar, as flexible, would bear to break it without being weakened by crushing (Art. 6.); b the breaking weight, as calculated by Articles 36 and 38.; y the real breaking weight.

Then, supposing a portion of the pillar, equal to what would just be crushed by the pressure d, to be taken away, we have $c - d =$ crushing strength of the remaining part, and $y - d$ the weight actually laid upon it. Whence $\frac{y-d}{c-d} =$ the part of this remaining portion of the pillar which has to resist crushing, $\therefore 1 - \frac{y-d}{c-d} = \frac{c-y}{c-d} =$ the part to sustain flexure.

But the strength of the pillar, if rectangular, may be supposed to be reduced by reducing either its breadth, or the calculated strength of the whole, to the degree indicated by the fraction last obtained. In the circle this mode is not strictly applicable; but we obtain a near approximation to the breaking weight, y, by reducing the calculated value of b in that proportion. We have, therefore,

$$c - d : c - y : : b : y,$$

$$\therefore cy - dy = bc - by,$$

whence

$$y = \frac{bc}{b + c - d}.$$

43. It has been shown (Art. 7.) that about one-fourth of the crushing weight was the utmost that a flexible pillar could be broken with, without the material being crushed, and the breaking weight reduced in consequence.

In the Table following, I will give the values of y, as deduced from all the pillars in Article 39, taking $d = \frac{c}{4}$; and, therefore, $y = \frac{bc}{b + \frac{3}{4}c}$. Before concluding this subject, I may, however, observe here, that there is a falling off in the strength of pillars, in consequence of the pressure, through all stages, from the smallest force necessary to break them to the largest, though this diminution does not become so obvious as to need correcting for, till the breaking weight is about one-fourth of the crushing weight. This reduction in strength will be shown from a comparison of the strengths of the thicker with the more slender pillars, the length being the same, the falling off always increasing with the diameter. This will be rendered evident from the decrease in the

values of n (Art. 23 to 25.). We may also see its effects in the comparison between the strengths of pillars 1·29 diameter, and between those of 1·56, in the abstract (Art. 5.). In each of these cases the pillars were of the same diameter nearly; but as the breaking weights were, from another cause, very different, the comparative strength was considerably lower than in the pillars of smaller diameter. This falling off in the strength of large pillars of equal lengths arises principally from this circumstance, that the strength to resist flexure increases as a power of the diameter somewhat under the fourth; and the resistance to crushing only as the second. Thus the pillars of greater diameter require a much greater load upon the same area of section to break them than those with smaller diameters. The position of the neutral line is therefore different, and the area of the part submitted to tension reduced; the pillar, therefore, becomes more flexible, and consequently weaker.

Some small part of this falling off in the strength of larger pillars might be accounted for by the softer quality of cast iron when cast in large masses; but other materials exhibit the same diminution of strength, and the cause above is sufficient to account for it.

Tabulated results of the formula $y = \frac{b\,c}{b + \frac{3}{4}c}$.

Diameter of pillar.	Length of pillar.	Value of b.	Value of c.	Breaking weight.	Calculated breaking weight from formula for y.
inches.	inches.	lbs.	lbs.	lbs.	lbs.
·50	12·1	8327	21559	7195	7328
·50	10·083	11353	21559	8931	8872
·50	7·5625	18515	21559	11255	11508
·50	3·7812	60155	21559	17468	16992
·777	20·166	16713	52064	15581	15604
·775	15·125	27005	51797	21059	21241
·785	12·1	41300	53142	24287	27043
·768	10·083	52096	50865	25923	29363
·777	7·5625	88547	52064	32007	36130
1·022	20·1666	44218	90074	31804	35631
1·000	15·125	66746	86238	40250	43797

The formula for y gives results agreeing moderately well with the strengths of pillars decreasing in length, both here and in the experiments on hollow pillars, as will be seen further on (Art. 52.).

Similar Pillars.

44. We have found the strengths of solid pillars of cast iron to vary as $\frac{d^{3\cdot76}}{l^{1\cdot7}}$ and $\frac{d^{3\cdot55}}{l^{1\cdot7}}$, nearly; when they are not shorter than from fifteen to thirty times the diameter, according as the ends are rounded or flat (Art. 36 to 38.).

In the research for these numbers I have been led to conclude, that, if the material had been incompressible, the 3·76 and 3·55 would each have become 4, and the 1·7 have been 2 (Art. 24. 33.). In that case the strength would have varied as $\frac{d^4}{l^2}$, which is

the ratio according to the theory of EULER. In similar pillars the length is to the diameter in a constant ratio. Calling the length $n\,d$, where n is a constant quantity, we have, in these different cases, the strength as

$$\frac{d^{3\cdot76}}{n^{1\cdot7}\times d^{1\cdot7}},\quad \frac{d^{3\cdot55}}{n^{1\cdot7}\times d^{1\cdot7}},\quad \frac{d^{4}}{n^{2}\,d^{2}}.$$

or, making the divisions, these become

$$\frac{d^{2\cdot06}}{n^{1\cdot7}},\quad \frac{d^{1\cdot85}}{n^{1\cdot7}},\quad \frac{d^{2}}{n^{2}}.$$

In the first of these cases, the strength varies as a power of the diameter somewhat higher than the square; in the second, somewhat lower; and in the third, as the square. We may therefore conclude, that in similar pillars the strength is nearly as the square of the diameter, or of any other linear dimension*: and as the area of the section is as the square of the diameter, the strength is nearly as the area of the transverse section. The experiments in Tables I. and II. give for the strength of similar pillars, powers of their linear dimensions as below.

	Diameters of pillars compared.	Length of pillars compared.	Breaking weight of pillars.	Powers of the dimensions.	
	inch.	inches.	lbs.		
Pillars with rounded ends.	·497 ·99	7·5625 15·125	5262 19752	1·908	Mean from the powers 1·865.
	·76 1·52	15·125 30·25	9223 32531	1·819	
	·99 1·97	30·25 60·5	6105 25403	2·057	
Pillars with flat ends.	·51 1·56	20·166 60·5	3830 28962	1·841	
	·50 ·997	30·25 60·5	1662 6238	1·9081	
	·51 1·02	15·125 30·25	6764 21844	1·6913	
	·50 1·022	10·083 20·166	8931 31804	1·8323	

In the Table above, the pillars compared were from models which were similar. I have, therefore, neglected slight deviations from similarity in the castings from these models. It appears then that the power of the lineal dimensions, according to which their strengths vary, is somewhat lower than the second.

45. If pillars be so formed as equally to resist being crushed (as shown in Art. 6.) by the breaking weight, they will be similar.

We have seen, that when pillars require a force to break them by flexure, which exceeds a certain portion of the force which would crush them without flexure, the pillar

* In deducing this conclusion, EULER remarks, that if, of two similar pillars of the same material, one be double the linear dimensions of the other, the larger will but bear four times as much as the smaller, though its weight is eight times as great. Berlin Memoirs, 1757.

sustains a considerable diminution in its power of resistance to fracture by flexure, in consequence of a partial crushing, or crippling of the material. Suppose $c\,d^2 =$ the crushing force of the pillar (d being its diameter), or that pressure which would cause rupture in it, if it were too short to break by flexure; and $n\,c\,d^2$ that part of this pressure, which is the utmost it would, as flexible, sustain without apparent crippling or crushing. Then, since the strengths to resist fracture by flexure in pillars, with both ends rounded and both flat, were $33379 \frac{d^{3\cdot76}}{l^{1\cdot7}}$, and $98922 \frac{d^{3\cdot55}}{l^{1\cdot7}}$ respectively (Art. 36. 38.), we have these quantities each equal to $n\,c\,d^2$, in the cases where short pillars are bearing the greatest weights they can sustain without any apparent crushing. Whence,

In pillars with rounded ends,

$$33379 \frac{d^{3\cdot76}}{l^{1\cdot7}} = n\,c\,d^2; \therefore\ l = \left(\frac{33379}{n\,c}\right)^{\frac{1}{1\cdot7}} \times d^{\frac{1\cdot76}{1\cdot7}};$$

In pillars with flat ends,

$$98922 \frac{d^{3\cdot55}}{l^{1\cdot7}} = n\,c\,d^2; \therefore\ l = \left(\frac{98922}{n\,c}\right)^{\frac{1}{1\cdot7}} \times d^{\frac{1\cdot55}{1\cdot7}}.$$

In the former of these cases, l varies somewhat faster than as the first power of the diameter, and in the second, somewhat slower; the two showing that, in the case of pillars equally loaded to resist crushing by the weight, the length to the diameter will be nearly in a constant ratio, or the pillars must be similar.

46. *Strength of long uniform Pillars, as dependent on their weight, the length being constant.*

In general, the strength varies as $\frac{d^m}{l^n}$; where d and l are the diameter and length; and m, n, the constants on which the strength depends. If the length be the same, the strength varies as d^m; and since the weight q of the pillar is as the square of the diameter or directly as the area, $d \propto q^{\frac{1}{2}}$; whence strength $\propto q^{\frac{m}{2}}$.

In pillars with rounded ends, $m = 3\cdot76$ $\therefore$ strength $\propto q^{\frac{3\cdot76}{2}} \propto q^{1\cdot88}$.

In pillars with flat ends, $m = 3\cdot55$ $\therefore$ strength $\propto q^{\frac{3\cdot55}{2}} \propto q^{1\cdot775}$.

These values were used in comparing the strengths of pillars of equal weight in Tables VI. and VII., and in other places.

It will be borne in mind that this proposition applies only to pillars above a certain length, as in Articles 36 and 38.

Hollow Pillars of Cast Iron, the Low Moor, No. 3.

47. The preceding observations have been made upon solid pillars only, as these, on account of their greater simplicity, are not only more easy to be cast than hollow ones, but present fewer difficulties in experimenting upon, and in deducing conclu-

sions. Solid pillars may be made as small as may be desired, but this is not the case with hollow ones. These require great care on the part of the moulder, and are consequently expensive. The first hollow pillars used in these experiments were cast in a horizontal position, but it was found impracticable in this way to keep the core in the centre of the casting, and therefore several of the first pillars were defective, in having one side much thicker than the other, as well as in having many air bubbles in them. These causes tended to diminish the strength of the pillars; and will account for some results in Table VIII. being lower than they otherwise would have been. The circumstance of some of these pillars being much thicker on one side than the other, gave me an opportunity of observing what falling off in strength arose from that cause, which is one to which cast-iron pillars, as used in practice, must be very liable, though not so much so as in these small experimental castings. I have therefore recorded these cases with the others, conceiving that they would be interesting to practical men; and it is gratifying to find that a matter, which would seem to destroy all confidence in a pillar, does not produce a great reduction in the strength. The cause seemed to be this: in almost every case, where pillars were much thicker, in the middle, on one side than the other, the thin side was that which was compressed: and as cast iron resists fracture by compression, with (on the average) about six times the force that it resists tension, the pillar seldom gave way by compression, and therefore bore nearly as much as it would have done if of equal thickness on both sides.

If the thick side of the pillar had become that which was compressed, which, for what I can see, was as likely as otherwise, the strength would probably have been much decreased. But the circumstance of the thin side having been so often the compressed one, must apparently have arisen from some general cause.

The difficulty of obtaining good uniform castings caused me to mention the circumstance to Mr. Fairbairn, who gave orders that all the future pillars should be cast vertically, and in dry sand; and this, together with great care, produced good castings, though the core could not always be kept in the centre towards the middle of the pillar.

The greatest length of pillar to which the apparatus was adapted, was about seven feet six inches; being a length half as much again as the greatest which was used in the experiments upon solid pillars. As, in the former experiments, the five-feet pillars had, by a mistake in the commencement, been made half an inch too long, my anxiety to avoid error, however slight, caused me to have the lengths of all the shorter pillars exact subdivisions of the longer; this entailed upon all the future experiments fractional lengths, which have been introduced into the hollow cylinders, in order to make them exact multiples of the former.

The length of the hollow pillars was therefore made 7 feet $6\frac{3}{4}$ inches; and it was my wish (which was always seconded by my liberal friend Mr. Fairbairn) that their diameters should be as varied as possible, both for theoretical purposes and the application of their results to practice. There were difficulties, however, which militated

against the execution of this desire. The external diameter of the pillars could not be reduced at pleasure; the internal must be large enough to admit a small hollow metallic tube, covered over with a thin stratum of loam, tied together with vegetable matter, the tube being perforated in every part with holes, and having its internal diameter large enough to allow the gas and steam to escape.

Through these causes, the smallest pillars which we could cast of the above length were $1\frac{3}{4}$ inch external diameter, and $1\frac{1}{5}$ internal. On the other hand, the experiments were bounded, in the largeness of the diameters, only by the power necessary to break the pillars. With Mr. FAIRBAIRN's apparatus (a single lever of which the greatest multiplier was 7 or $7\frac{1}{4}$) I had never conceived it safe to apply more pressure than about 18 tons; and when an additional lever was added to this, the greatest weight used was $22\frac{1}{2}$ tons. This was in one of the experiments on hollow pillars.

Through these causes, the variation in the diameters of the longest hollow pillars was necessarily small, particularly in those with flat ends, of which the largest external diameter was about $2\frac{1}{4}$ inches, this requiring upwards of 18 tons to break it. In this, the length of the pillar was 40 times the external diameter, and therefore might be considered as representing only the longest and slenderest pillars used in practice. In this matter, however, I was not without a resource, it having been discovered, in the early part of these inquiries, that the ratio of the strengths of solid pillars with rounded ends, to those with flat ones, was nearly constant, and about as one to three; and this was the case with all pillars, from those of the greatest length, as compared with the diameter, down to a certain ratio of length, considerably shorter than we could command power to break, provided the ends of the pillar were flat.

It was therefore determined to pursue the experiments upon these hollow pillars (putting solid hemispherical caps upon the ends), increasing the diameters by small degrees (the length being the same), till I could break them no further.

In this way I was enabled to vary the external diameters from 1·74 to 3·36 inches, the former having sunk with 5711 lbs., and the latter with 50477 lbs., or $22\frac{1}{2}$ tons. In the largest pillars of this series, experiments 15 and 16., Table VIII., the length was reduced to twenty-seven times the diameter. To pursue the matter further, it now became necessary to reduce the length of the pillars, because when that was done the internal diameter could be reduced, and thus the external might be cast so small that the strength of the metal in the pillar would be overcome by the power which I had at command. Several of these pillars were, at Mr. FAIRBAIRN's suggestion, turned outside and bored within, to make their dimensions as uniform as possible. By adopting pillars of smaller size than before, I was now enabled to render them as short, compared with the diameters, as I wished, and thus obtain specimens of most of the varieties used in practice. (See Table X.)

Tables VIII. and IX. contain the results of the experiments upon the longest hollow uniform cast-iron cylinders; with an experiment or two in each, upon solid cylinders of the same length to compare with them. The cylinders were always cast

with great care, and were either perfectly straight, or they were rendered so in the lathe, where their ends were turned flat and parallel. The pillars intended to be used with rounded ends, were made about two inches shorter than the rest, or 7 feet $4\frac{3}{4}$ inches long, in order to allow room for the rounded caps upon the ends, as before mentioned, the whole length of the pillar with caps being 7 feet $6\frac{3}{4}$ inches.

Description of the Tables.

48. The first column in the Tables on hollow cylinders contains the number of the experiments only, and needs no explanation. The second column, in giving the internal diameter of the pillar, gives it as calculated as well as measured. This was done because there was great difficulty in measuring correctly the internal diameters, which were in some degree irregular. The weight of a solid pillar of a certain diameter, and the given length, being known, from a mean of three experiments, and the weights of the hollow cylinders, together with their external diameters, being also known, from these data, the internal diameters of all the cylinders were computed; and a mean between the calculated and the observed results was taken for the internal diameter. This extra trouble was taken, as I was desirous of avoiding every possible inaccuracy in experiments which, from the expense and labour attached to them, were not likely to be soon repeated. The third and fourth columns of Tables VIII. and IX. contain the deflections caused by different weights laid upon the pillar, and the weights themselves. The fifth column contains what is denominated the breaking weight, but which means only that weight which was sufficient to overcome the greatest resistance of the pillar. This was frequently attained when a long pillar had been bent through a space perhaps not exceeding its radius. From the first experiment on long hollow pillars with rounded ends, it was evident that so little flexure of the pillar was necessary to overcome its greatest resistance (and beyond this a smaller weight would have broken it), that the elasticity of the pillars was very little injured by the pressure, if the weight was prevented from acting upon the pillar after it began to sink rapidly, through its greatest resistance being overcome.

By attending carefully to this point, the pillar was frequently saved uninjured for another experiment. The small flexure that it had got was straightened, and the rounded ends being removed, the pillar, with its flat ends well bedded against the crushing surfaces, was broken, and the results in Table IX. were recorded. The sixth columns in Tables VIII. and IX. contain the strength of a solid pillar, 7 feet $6\frac{3}{4}$ inches long, and one inch diameter, as calculated on a theoretical supposition* mentioned further on. This value is always calculated for the full length, 7 feet $6\frac{3}{4}$ inches, though some of the pillars in Table IX. were two inches shorter than that. This reduction, as well as others of a similar nature, was made on the supposition that the strength of pillars of the same thickness varies inversely as the 1·7th power of the length, as was found to be the case in solid pillars.

It had been shown, Table VI., that a solid cylindrical pillar, with rounded ends,

* Poisson, Mécanique, 2nd edition, vol. i. pp. 619 and 620.

and an enlarged middle, was stronger than a uniform pillar of the same weight. There seemed little doubt that the same would be the case in hollow pillars, or that a hollow pillar with an enlarged middle, having the metal everywhere of uniform thickness, would be stronger than a hollow uniform pillar of the same weight and thickness of metal. To form these pillars with much regularity in small experimental castings, would have been difficult; but some idea of the influence of increasing the strength of the middle, might be obtained by reducing the thickness of the uniform pillars, near to the ends, leaving the middle the same. The pillars, in experiments 7 and 8. of Table VIII., in which the ends were rounded, together with one or two others, were reduced in this manner. The pillar was reduced to half its thickness of metal, in a band two or three inches broad, near to each end, and to three-fourths of its thickness, in a similar band, about half the distance between the middle and each end. These pillars did not break in the reduced parts; and from the results it would seem doubtful, whether the strength had been in any degree lessened by the partial reduction of thickness. If this reduction had been carried uniformly on, decreasing from the ends to the middle of the pillar, leaving it the same there, the pillar would but have had three-fourths of the metal in it, and its strength would, perhaps, have been but little decreased.

Of the first ten pillars in Table IX., being those with their ends flat, one half were reduced by having portions cut away from each, as before described. These pillars never broke in the reduced parts first, except in one instance, and that was where the thickness, half-way between the middle and the ends, had been reduced somewhat more than one-fourth, which was the quantity ordered to be taken away from that part.

The calculated values in Table IX., taking a mean from the five reduced pillars, give 3060; and from the five which were not reduced, 3308; the latter being $\frac{1}{13}$ stronger than the former, leaving it doubtful whether reducing the strength toward the ends, in pillars with flat ends, effects any saving of metal. The experiments in Table VII., upon solid pillars, with discs upon the ends, show a saving, however, when the strength in the middle is much greater than toward the ends. The short pillars in Table X. will be referred to further on.

In Table XI., the first two pillars were from the same model. They were about $2\frac{1}{4}$ inches diameter in the middle, and $1\frac{3}{4}$ inch near to the ends, which had strong discs upon them turned flat, to give them a firm bearing; the pillars were hollow, as before. The weight of the former was 36 lbs., and of the latter $36\frac{1}{4}$ lbs., and their breaking weights 28667 and 27491, mean 28079 lbs. From a mean between experiments 5, 6, 7, Table IX., it appears that a hollow uniform pillar 2·02 inches external diameter (the mean of the above nearly), and weighing $36\frac{1}{2}$ lbs., required 30518 lbs. to break it. This seems to show that the uniform cylinder is the stronger*.

* This conclusion is in agreement with what may be deduced from EULER's theoretical values of the strengths of uniform cylindrical solid pillars, and of those in the shape of a truncated cone (Berlin Memoirs, 1757); his formulæ for these being $P = \frac{a^2 D^4}{A^2 d^4} \cdot p$, and $P' = \frac{a^2 D'^2 E^2}{A^2 d^4} \cdot p$.

These values are to one another, *cæteris paribus*, as D^4 to $D'^2 E^2$; where D is the diameter of the uniform

The third and fourth pillars in Table XI., are two hollow pillars $2\frac{1}{4}$ inches external diameter at bottom, and $1\frac{3}{4}$ at top nearly. They tapered regularly from the bottom to the top, where there was a disc to increase the stability of fixing. The mean weight of these was 36·8 lbs., and the mean breaking weight 29488 lbs. But we have seen above, that a uniform hollow pillar 2·02 inches diameter, and $36\frac{1}{2}$ lbs. weight, would require 30518 lbs. to break it. Therefore these pillars, like the former ones, are weaker than uniform hollow pillars. Experiment 5. was with a uniform pillar having rounded ends. The section was of the form , three inches across; the pillar weighed 62 lbs., and was broken with 17578 lbs. But a uniform hollow pillar 3·01 inches diameter, and $50\frac{1}{4}$ lbs. weight (Experiment 13., Table VIII.), required 26707 lbs. to break it. Whence we see, that pillars of this form, which is nearly that of the connecting rod of a steam-engine, are very weak compared with uniform hollow cylindrical ones; the strength being as the 1·88 power of the weight nearly (Art. 46.), the hollow cylinder, if of the same weight, 62 lbs., as the pillar above, would have required 39645 lbs., or more than twice 17578, to break it.

Experiment 6. is the result from a model differing from the last in having the section of the form . This was considerably stronger than the preceding, but much weaker than a hollow cylinder of the same weight.

Experiments 7 and 8. show the strength of solid pillars when the pressure is applied to some point intermediate between the centre and the circumference; and exhibits a great falling off, from what it would have been, if the pressure had been through the axis, as in pillars from the same model in Table I.

Experiment 9. is on a pillar so fixed that the pressure is in the direction of the diagonal; and comparing its strength with that of one of the same dimensions, with rounded ends, in Table I., it is nearly the same. The strength is, therefore, only one-third of what it would have been, if the pillar had been erect, with its ends well supported, as in Table II.

The long pillars referred to in this paper will, I conceive, comprise most of the forms at present used in practice, including the best of those employed by the ancients. With respect to pillars so short as to break by crushing, experiments on twenty-eight species of timber are given at the end of the paper; and on this subject I would beg to draw the attention of the reader to my Report on cast iron, in the Sixth Volume of the British Association, before referred to. The experiments upon crushing, by M. Rondelet (Art de Bâtir), and by Mr. George Rennie, will be known to most of my readers.

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pillar, and D′, E the diameters of the two ends of that in the form of a truncated cone. But if we compute the diameter of a uniform cylindrical pillar of the same length and solid content as one with unequal diameters, we shall find the uniform pillar stronger than the other, and the more so according as the inequality of the diameters of the latter is greater.

Computed Strength of Hollow Cast-Iron Cylindrical Pillars.

49. In solid cylinders of the same material, the strength to resist breaking varies according to some constant power n of the diameter; the length of the pillar and form of the ends being the same. Thus, strength $\propto D^n$, where D is the diameter. In cylinders with rounded ends, my experiments give $n = 3{\cdot}76$ nearly; and in those with flat ends, $n = 3{\cdot}55$ nearly.

According to the theory of EULER, the strength of a hollow cylinder, to resist incipient flexure, varies as the difference of the fourth powers of the radii, or of the diameters, external and internal, or as $D^4 - d^4$*, where D and d are the external and internal diameters†.

It appeared probable that this ratio, modified by changing the indices to the results of the experiments upon solid pillars, might agree with those upon hollow ones. This was tried, and, with great satisfaction, found to answer.

We have, therefore,

Strength as $D^{3{\cdot}76} - d^{3{\cdot}76}$, in pillars with rounded ends;

Strength as $D^{3{\cdot}55} - d^{3{\cdot}55}$, in pillars with flat ends.

To determine the strength of a solid pillar of a given length and one inch diameter, calling that strength x, and putting w for the breaking weight of a hollow pillar of the same length whose external and internal diameters are D and d, we have

$$D^{3{\cdot}76} - d^{3{\cdot}76} : 1^{3{\cdot}76} - 0^{3{\cdot}76} :: w : x.$$

Whence

$$x = \frac{w}{D^{3{\cdot}76} - d^{3{\cdot}76}}, \text{ for rounded ends.}$$

$$\therefore x = \frac{w}{D^{3{\cdot}55} - d^{3{\cdot}55}}, \text{ for flat ends.}$$

By these formulæ, the strength of a solid cylinder, 1 inch diameter and 7 feet $6\frac{3}{4}$ inches long, has been calculated from all our longest hollow cylinders, and inserted with the results of the experiments from Tables VIII. and IX.

The value of x thus obtained from each of the experiments, must, it is evident, be nearly constant, if the assumption respecting the indices be correct.

I have extracted these values from the different experiments and given them in the following Table, and also a fraction indicating the magnitude of the difference between the result from each experiment and the general mean, and consequently the error which would arise from calculating by the general mean.

* POISSON'S Theorem gives P the strength, $= \frac{\pi^3 a (g'^2 + g^2)(g'^2 - g^2)}{4 l^2}$, where $l =$ the length, $\pi = 3{\cdot}1416$, $a =$ a quantity, which is equivalent to the modulus of elasticity in weight per unit of section; and g', g the external and internal semi-diameters. But $(g'^2 + g^2)(g'^2 - g^2) = g'^4 - g^4$; and all the other quantities are constant, according to the assumption in the text.

† POISSON, Mécanique, vol. i. p. 620.

Pillars with rounded ends.		Pillars with flat ends.	
Values of x.	Error from using general mean.	Values of x.	Error from using general mean.
834·37	$+\frac{1}{8\cdot4}$	2973·70	$+\frac{1}{13\cdot9}$
933·13	$-\frac{1}{2522}$	3031·50	$+\frac{1}{19\cdot5}$
826·20	$+\frac{1}{7\cdot7}$	2968·70	$+\frac{1}{13\cdot6}$
903·28	$+\frac{1}{30\cdot6}$	3586·90	$-\frac{1}{8\cdot9}$
808·21	$+\frac{1}{6\cdot5}$	3573·80	$-\frac{1}{9\cdot2}$
1041·70	$-\frac{1}{9\cdot5}$	3290·30	$-\frac{1}{31\cdot6}$
917·62	$+\frac{1}{60\cdot6}$	3214·80	$-\frac{1}{112\cdot8}$
996·24	$-\frac{1}{15\cdot6}$	2988·30	$+\frac{1}{15\cdot1}$
1123·50	$-\frac{1}{5\cdot8}$	3099·00	$+\frac{1}{35\cdot1}$
989·95	$-\frac{1}{17\cdot3}$	3115·50	$+\frac{1}{44}$
949·48	$-\frac{1}{56\cdot7}$	3207·70	$-\frac{1}{149\cdot9}$
1059·50	$-\frac{1}{8\cdot3}$		
819·46	$+\frac{1}{7\cdot2}$		
895·02	$+\frac{1}{23\cdot7}$		
880·11	$+\frac{1}{16\cdot7}$		
1026·60	$-\frac{1}{10\cdot9}$		
927·86	$+\frac{1}{189\cdot3}$		
1005·30	$-\frac{1}{13\cdot8}$		
784·90	$+\frac{1}{5\cdot3}$		
General mean 932·76.		General mean 3186·38.	

50. The last value of x, in pillars with rounded ends, was from a pillar so short as evidently to have been a little crushed by the breaking weight, a matter which we have seen weakens a pillar (Art. 6.).

If the results from this experiment had been omitted, the general mean would have been increased from $932\frac{3}{4}$ to 941, and the fractional errors somewhat lessened. Another cause of irregularity in some of these pillars has been before alluded to (Art. 47.).

The mean values of x, above, represent the strength of a solid pillar one inch diameter, and 7 feet $6\frac{3}{4}$ inches long, as obtained by calculation, on the preceding principles, from pillars whose length was thirty times the diameter, or upwards, in those with flat ends, and half that length or more, in those with rounded ends.

It was shown (Art. 31, 32.), that when the lateral dimensions of pillars are the

same, the strength varies inversely as the 1·7th power of the length, as a sort of general average. Adopting this number, and calculating by it the strength of a pillar one foot long, from the strength of one 7 feet $6\frac{3}{4}$ inches long, we have the strength of solid pillars one foot long and one inch diameter,

In pillars with rounded ends 29074 lbs. = 12·979 tons;
In pillars with flat ends . 99318 lbs. = 44·338 tons.

The mean strengths of pillars one foot long and one inch diameter, as obtained (Art. 35 to 37.) from the experiments on solid pillars, are 33379 lbs. and 98922 lbs. respectively; which, in pillars with flat ends, is pretty nearly in agreement with that above. But the defective specimens in the first hollow pillars, with rounded ends, has caused the number to be 29074, whilst the solid cylinders gave 33379 lbs.

51. Taking the numbers above, as deduced from the hollow pillars, we have for the strength of hollow cylindrical pillars in general,

$$29074\,\frac{D^{3\cdot76}-d^{3\cdot76}}{l^{1\cdot7}},\text{ in pillars with rounded ends;}$$

$$99318\,\frac{D^{3\cdot55}-d^{3\cdot55}}{l^{1\cdot7}},\text{ in pillars with flat ends.}$$

In using these formulæ, which answer for solid cylinders when $d=0$, it must be borne in mind that they do not apply to pillars whose length is less than about thirty times the external diameter when the ends are flat, nor to those of less than half that length when they are rounded.

Short hollow Pillars.

52. When pillars are shorter than as above, we have seen (Art. 6.) that there is a falling off in their breaking strength, on account of a change being produced in the material through the great weight necessary to break them. This falling off has been attempted to be accounted for (Art. 41.), and a formula given by which the strengths of pillars, however short, could be calculated from the theorems used for long pillars, by means of the crushing strength of the body, the latter being shown to be an element in the resistance of pillars to fracture by flexure.

This formula for short pillars (Art. 43.) is

$$y=\frac{b\,c}{b+\frac{3}{4}c},$$

where b is the strength of the pillar, as calculated by the rules for long pillars (Art. 51.), and c the weight which would be required to crush the pillar without flexure.

Table X. contains the results of experiments upon thirteen pillars varying in length from twenty-four times the external diameter down to less than eight times. In this table, the deflections were not observed, as they were very small, and required much care; and there was considerable danger in observing them, for the pillar usually broke, with violence, into many pieces. They were made, however, with great care:

the castings were good, and were, as before remarked, frequently bored within and turned on the outside, to ensure uniformity as much as possible.

The values of b, c, and y, are calculated and inserted in the Table. I will extract from it the real and calculated breaking weights, that the reader may see how far they agree.

Real breaking weight.	Calculated breaking weight.	Difference in parts of real breaking weight.
33679	32331	$-\frac{1}{24\cdot9}$
32867	31790	$-\frac{1}{30\cdot5}$
35302	36501	$+\frac{1}{29\cdot4}$
31195	28764	$-\frac{1}{12\cdot8}$
30383	30291	$-\frac{1}{33\cdot0}$
41751	40128	$-\frac{1}{25\cdot7}$
27135	29449	$+\frac{1}{11\cdot72}$
25511	26191	$+\frac{1}{37\cdot5}$
25105	26273	$+\frac{1}{21\cdot49}$
26729	27364	$+\frac{1}{42\cdot0}$
27135	30863	$+\frac{1}{7\cdot27}$
37285	40257	$+\frac{1}{12\cdot2}$
34037	31750	$-\frac{1}{14\cdot8}$

Power of Cast-iron Pillars to resist long-continued Pressure.

53. In all the previous experiments, the pillars were broken without any regard to time, and an experiment seldom lasted longer than from one to three hours. There might, therefore, be considerable doubt upon the minds of many persons whether the results obtained would be consistent with those which would arise from long-continued pressure.

At my suggestion, therefore, Mr. Fairbairn had the apparatus (fig. 2. Plate XIV.) erected, by which pillars might be permanently loaded. Four pillars from the same model were used; they were each placed vertically upon a horizontal iron plate on the ground, and pressed upon at the top by means of a pin, kept vertical by passing through an iron collar surrounding a hole bored in the horizontal timber at the top of the frame. The bottom ends of these pins were slightly concave, that they might press, without slipping, upon the top of the pillars, which were rounded at the ends, whilst the top of the pin was rounded, that it might be acted upon uniformly by the square flat plate which rested upon it, and which by means of the suspending rods at the corners supported the weight.

Mr. FAIRBAIRN intends that the results of these experiments shall continue to be observed in conjunction with his interesting experiments upon the effect of long-continued pressure upon horizontal bars.

The results obtained up to the present time are below, as taken from Mr. FAIRBAIRN's register.

Uniform pillars of Low Moor Iron, No. 3, each six feet long and one inch diameter, rounded at the ends, and placed vertical with the following weights on each.

Date of observation.	Temperature FAHR.	1st Experiment. Weight of pillar 14 lbs. 13 oz. Permanent load 448 lbs.	2nd Experiment. Weight of pillar 14 lbs. 13 oz. Permanent load 784 lbs.	3rd Experiment. Weight of pillar 14 lbs. 11 oz. Permanent load 1120 lbs.	4th Experiment. Weight of pillar 14 lbs. 13 oz. Permanent load 1456 lbs.
1839.		Deflections. inch.	Deflections. inch.	Deflections. inch.	Deflections. inch.
July 5.	72°	·018	·03		
6.	68			·215	·250
8.	69	·01	·02	·230	·275
16.	63	·01	·023	·235	·310
23.	64	·01	·02	·235	·320
Aug. 15.	63	·01	·023	·235	·395
Nov. 7.	50	·01	·03	·24	·825
Dec. 9.	39	·01	·025	·243	·955
1840.					
Feb. 14.	50	·01	·025	·245	Broke about 19th or 20th December, 1839.
April 27*.	63	·01	·025	·365	
June 6.	61	·01	·025	·380	
Aug. 3.	74	·01	·030	·395	
Sept. 14.	55	·01	·025	·380	

A pillar of the same dimensions, and cast at the same time as those above, was broken with 1500 lbs. laid gradually upon it. Hence, supposing the strengths to be the same, we see that the pillar in the fourth experiment above, had borne $\frac{1456}{1500}$, or $\frac{97}{100}$ of its breaking weight, between five and six months; though the deflection had been regularly increasing from the first. I would therefore infer, that the short or long time employed in making an experiment has little influence upon its result.

In concluding my inquiries into the strength of cast-iron pillars, I shall give the tensile, compressive, and transverse strengths, together with the specific gravity of the iron made use of, to serve as data for other researches.

Tensile Strength of Low Moor Iron, No. 3, or resistance which it would offer to a direct force tending to tear it asunder.

54. The castings were of the same form as were used for this purpose in my Report on the Strength of Cast Iron†. The form of the section torn asunder was ; and the force was in a line perpendicular to this, and passing through the centre. The

* The experiments of this and the subsequent dates were inserted during the printing of the paper.

† Transactions of the British Association, vol. vi.

ends of the castings were made very strong and rigid, and they were perforated, so that links passing through them, to tear the casting asunder by, should act directly through the axis.

	square ins.				
1st exp. area of section	1·5330	Mean 1·540	Breaking weight	21219	Mean 22385,
2nd exp. area of section	1·5470		Breaking weight	23551	

or strength, per square inch, 14535 lbs. = $6\frac{1}{2}$ tons nearly.

Compressive Strength of the Iron used above, or that which would crush it without Flexure.

55. The first four experiments on this matter were on rectangular masses, cut out of the pieces torn asunder, the sides being rendered smooth and perpendicular to each other. The other are from the shortest cylinders in Table II.

No. of Experiment.		Area of section in inches.	Height of specimen.	Crushing weight.		Crushing weight per square inch.
		sq. in.	inches.	lbs.		lbs.
1	Rectangles.	1·00 × ·25 = ·25	·50	25923	Mean 25923	103692
2		1·00 × ·25 = ·25	·50	25923		
3		1·00 × ·25 = ·25	1·00	24355	Mean 24551	98204
4		1·00 × ·25 = ·25	1·00	24747		
5	Cylinders ·52 inch diameter.	·21237	1·00	23963	Mean 24616	115911
6		·21237	1·00	24747		
7		·21237	1·00	25139		
8		·21237	2·00	23035	Mean 22867	107675
9		·21237	2·00	22699		

When bodies are crushed, as above, they give way by a wedge sliding off in an angle dependent upon the nature of the material; and in cast iron the height of this wedge is about $1\frac{1}{2}$ the diameter, or thickness of the base of the wedge. If the body to be crushed is shorter than would be sufficient to admit a wedge of the full length to slide off, then it would require more than its natural degree of force to crush it; because the wedge itself must either be crushed, or slide off in a direction of greater difficulty. If, on the other hand, the height of the body to be crushed be much greater than the length of the wedge, then the body will sustain some degree of flexure, and fracture will be facilitated in consequence.

To determine the crushing strength of the iron above used, I shall, for the reasons just given, take the results from those specimens which are but little longer than the wedge.

Taking, therefore, a mean from the results of the rectangles half an inch high, and the cylinders one inch, we have,

From the former	103692	Mean 109801 lbs. = 49·0183 tons per square inch.
From the latter	115910	

Whence we obtain 86238 lbs. = 38·499 tons, for the weight which would crush a

short cylinder one inch in diameter, since the crushing force is as the area of the section, as I have shown*.

Transverse Strength of the same Iron (*Low Moor, No.* 3.).

56. To determine this, the results from two square bars placed upon horizontal supports, four feet six inches asunder, and broken by weights suspended from the middle, are given below.

First Experiment. Depth of bar 1·002 inches. Breadth of bar 1·004 inches.			Second Experiment. Depth of bar 1·005 inches. Breadth of bar ·996 inches.		
Weight.	Deflection.	Deflection. Load removed.	Weight.	Deflection.	Deflection. Load removed.
lbs.	inch.	inch.	lbs.	inch.	inch.
28	·069	·002 ?	28	·060	
56	·145	·006	56	·141	·005
112	·307	·020	112	·321	·030
224	·677	·07	224	·725	
336	1·143	·17	336	1·230	
392	1·413	·26	392	1·550	
448	1·742	·38	448	1·910	
461	broke.		473	broke.	

The weights were increased toward the last by about 7 lbs. at a time. The modulus of elasticity in lbs. per square inch, obtained from the deflection caused by 112 lbs. is,

$$\left.\begin{array}{ll}\text{From the first experiment} & 14{,}251{,}950 \\ \text{From the second experiment} & 13{,}585{,}530\end{array}\right\} 13{,}918{,}740 \text{ Mean.}$$

Specific Gravity of the same Iron.

57. This, from the results of six experiments, is as below:

7122 from cylinders three-fourths of an inch diameter.
7135 from cylinders three-fourths of an inch diameter.
7034 from cylinders three-fourths of an inch diameter.
7090 from cylinders half an inch diameter.
6977 from cylinders half an inch diameter.
6952 from a hollow cylinder.

Mean 7052.

Pillars of Wrought Iron, Steel, and Timber.

58. The experiments upon these, which are given in Tables XII. and XIII., are similar in their objects to those upon cast iron, but much more limited in their extent. They were made, indeed, principally to confirm views which presented themselves during the experiments upon cast-iron pillars.

* Transactions of the British Association, vol. vi. before alluded to.

The following abstract contains the lengths, lateral dimensions, and breaking weights of the three kinds of pillars tried.

	Length.	Pillars with both ends rounded.		Pillars with one end flat, and the other rounded.		Pillars with both ends flat.	
		Diameter.	Breaking weight.	Diameter.	Breaking weight.	Diameter.	Breaking weight.
Wrought iron.	inches.	inch.	lbs.	inch.	lbs.	inch.	lbs.
	$90\frac{3}{4}$	1·017	1808	1·02	3355	1·02	5280
	$60\frac{1}{2}$	1·015	3938	1·03	8137	1·02	12990
	$30\frac{1}{4}$	1·015	15480	1·015	21355	1·015	23371
	$30\frac{1}{4}$	1·015	15480	1·015	21187 disc.	1·015	25387 disc.
	$15\frac{1}{8}$	1·005	23535	1·005	26227	1·005	27099
Steel.	29·95	·87	10516	·87	20315	·87	26059
Timber.	$60\frac{1}{2}$	Side of square. 1·75	3197	Side of square. 1·75	6109	Side of square. 1·75	9625

The pillars in each horizontal line in this abstract are of equal length, and nearly of equal diameter, the only essential difference being in the form of their ends. Taking the longest pillars, those of $90\frac{3}{4}$ and $60\frac{1}{2}$ long, the strengths are

	Ends rounded.	Ends rounded and flat.	Ends flat.
In iron . . .	1808	3355	5280
In iron . . .	3938	8137	12990
In timber . .	3197	6109	9625

The strengths in these three cases are, therefore, as one, two, three nearly, the middle being an arithmetical mean between the other two. These results are a corroboration of those from cast-iron pillars (Art. 16.). We see, too, from the wrought-iron pillars $30\frac{1}{4}$ and $15\frac{1}{8}$ inches long, as well as from those of steel, which appear to have been slightly crushed by the weight (Art. 6.), that this ratio one, two, three, does not obtain in short pillars, but that the difference of the strengths of the three forms of pillars becomes less, according as the number of times which the length of the pillar exceeds the diameter decreases. But whatever may be the ratio of the strengths of pillars with rounded ends to those with flat ones, the strength of those with one end rounded and one flat, is nearly an arithmetical mean between the strengths of the other two, as will be seen from the numbers below.

Mean from strengths of pillars, with both ends rounded and both flat.	Strengths of pillars, one end rounded and one flat.
lbs.	lbs.
3544	3355
8464	8137
19426	21355
20434	21187
25317	26227
18287	20315
6411	6109

59. We have seen in the preceding abstract from Table XII., that the longest wrought-iron pillars of the three classes there considered, have their strengths as one, two, three nearly; but that the shorter pillars, those of $30\frac{1}{4}$ inches long, or less (the diameter being 1·015 inch nearly), differ much less in their relative strengths. These in the $30\frac{1}{4}$ inch pillars being

Ends round.	Ends round and flat.	Ends flat.
15480	21355	23371
15480	21187	25387

The second and third of these numbers in either series are much less than twice and three times the first number, which was a mean from three experiments.

We see, therefore, that these second and third numbers are the results from pillars which were affected by some cause that did not reduce the strength of the pillars of equal diameter and length, which were broken by the first weight, 15480 lbs. For it will be seen further on, that this first weight was as great as is in accordance with the results from the longer pillars.

60. To determine the cause of this anomaly, which had first been exhibited in the experiments upon cast iron, and afterwards was found to take place in wrought iron, steel, and timber, I felt to be very difficult. The experiments upon cast iron offered no satisfactory explanation of the matter, for it usually changes but little in form before fracture. Wrought iron appeared, therefore, to be a better material for exhibiting the cause.

Three small cylinders of the same iron as that in Table XII. were therefore formed. They were each exactly $2\frac{1}{2}$ inches long and ·62 inch diameter, their ends being turned flat and perpendicular to the axis; they were subjected to compression with their ends well-bedded, and after bearing certain weights they were tried by a gauge which had exactly fitted them before. The results are in the table below.

No. of Experiment.	Weight laid on.	Weight per square inch.	Reduction in length.	Remarks.
	lbs.	lbs. tons.	inch.	
1	6222	20610 = 9·2	·0 ?	No alteration, or, if any, extremely small.
1	7342	24319 = 10·9	·015	
1	8462	28028 = 12·5	·02	Diameter sensibly increased, cylinder not bent.
2	12444	41218 = 18·4	·04	Diameter increased from ·62 to ·63 inch, slightly bent.
3	18667	61830 = 27·6	·16	Bent a little, diameter increased to ·65 inch.

We see, from the above, that 6222 lbs. produced no sensible change in the cylinder ·62 inch diameter, but that 7342 lbs. and 8462, particularly the latter, which was equal to $12\frac{1}{2}$ tons per square inch, produced very obvious changes.

Since the resistance to crushing are as the areas, the former of these two values would give 19677 lbs., and the latter 22679 lbs., for the pressures which would produce an equal alteration in a bolt 1·015 inch diameter; which is the diameter of the

bars upon which most of the experiments on wrought iron were made. But we have seen (Art. 59.) that 21187 lbs., and 21355 lbs. produced a great change in the strength of the pillars $30\frac{1}{4}$ inches long; it was, therefore, obvious that incipient crushing was the cause of the great falling off in the strengths of these pillars, and indeed of those in all the preceding experiments (Art. 6.); and a knowledge of this fact prepared me for attempting to adapt the formula used for long pillars to the case of short ones (Art. 41.).

61. It was shown (Art. 12.) from the experiments upon cast iron, that a uniform pillar, with its ends flat, had the same strength as one of equal lateral dimensions, and half the length, with the ends rounded. Making a similar comparison between the wrought-iron pillars, in Table XII., or in the preceding abstract (Art. 58.), we obtain an approximation to the same result; and that even when the pillars, from their shortness, are much crushed by the breaking weight, which we have seen was the case with the shortest wrought-iron pillars.

Strength of Pillars of Wrought Iron and Timber, as dependent upon their dimensions.

62. On this subject I have laboured under considerable difficulty, for want of experiments of a sufficiently comprehensive character to furnish all the data that were required. The complete determination of the constants for the purpose would require experiments upon pillars, whose lengths, compared with their diameters, were nearly as various as those upon cast iron in this paper.

I trust, however, that the experiments in Tables XII. and XIII., and the analogies derived from the results on cast iron, will supply the deficiencies so far, as, in some measure, to meet the wants of practice.

Length.—To find the inverse power of the length to which the strength of wrought-iron cylindrical pillars, of a constant diameter, is proportional, I shall follow the mode used for cast-iron pillars (Art. 26. 27.); and confining the inquiry to pillars sufficiently long not to have been crushed (Art. 6. 60.) with the breaking weight, we have as below:

Lengths of which the strengths are compared.		Description of pillar.	Inverse power of the length.	
ft. ins.	ft. ins.			
7 $6\frac{3}{4}$	2 $6\frac{1}{4}$	Rounded ends.	1·9546	Mean 2·0316.
7 $6\frac{3}{4}$	5 $0\frac{1}{2}$	Rounded ends.	1·9199	
7 $6\frac{3}{4}$	5 $0\frac{1}{2}$	Flat ends.	2·2203	

Whence it appears that the strength is as the inverse square of the length nearly.

Diameter.—To find the power of the diameter, or of the side of a square, to which the strength of wrought-iron pillars of a given length is proportional. For this purpose we have, from Table XII., the following results:

In the pillars 2 feet $6\frac{1}{4}$ inches long, rounded at the ends, from a mean between three experiments, one of ·52 inch diameter had its greatest resistance overcome with

1260 lbs; and from a mean between three other experiments, a pillar of the same length and 1·015 inches diameter had its greatest resistance overcome by 15480 lbs.

Since $\frac{1\cdot015}{\cdot52} = \frac{1\cdot9519}{1}$, we have, putting n for the power, $1^n : 1\cdot9519^n :: 1260 : 15480$.

Whence reducing, and taking the logarithms, we obtain $n = 3\cdot75073$, which is nearly the same as the mean from the cast-iron pillars with rounded ends (Art. 23.).

Computed Strength of Wrought-iron Pillars long enough not to be crushed, as in Art. 59. *and* 60., *by the weight which would overcome their greatest resistance.*

63. We have seen, from the last article, that the strength of cylindrical pillars whereof the length is constant, varies nearly according to the same power of the diameter as that in cast-iron ones. But where the diameter is constant, the strength is inversely as the square of the length, nearly.

Whence if, in round or square pillars, we put d for the diameter, or side of the square, l for the length, b the breaking weight, a, a' constant quantities, and adopt the powers of d, which were used for cast iron, we shall have (Arts. 36. 38.)

$$b = \frac{a\,d^{3\cdot76}}{l^2},$$

in pillars with rounded ends;

$$b = \frac{a'\,d^{3\cdot55}}{l^2},$$

in pillars with flat ends.

From the pillars in Table XII. we obtain as below:

	ft.	ins.		lbs.
From those	7	$6\frac{3}{4}$	long, with rounded ends, $a =$	97049
From those	5	$0\frac{1}{2}$	long, with rounded ends, $a =$	94648
			Mean value of $a =$	95848

	ft.	ins.		lbs.
From those	7	$6\frac{3}{4}$	long, with flat ends, $a' =$	281464
From those	5	$0\frac{1}{2}$	long, with flat ends, $a' =$	307770
			Mean value of $a' =$	299617

Here the length is taken in feet, and the diameter in inches.

Timber.

64. To find n the power of the diameter, or of the side of a square, to which the strength is proportional, the length being constant, and the pillar so long as not to be crushed (Arts. 59. 60.) with the breaking weight. For this purpose Table XIII. will supply us with the results of six experiments upon pillars of Dantzic oak, 46·1 inches long. Whence it appears that a pillar 1·02 inch square was broken with a mean weight = 1754 lbs., and one 1·50 inch square with 7888 lbs.

Here $\frac{1 \cdot 50}{1 \cdot 02} = \frac{1 \cdot 470}{1}$, $\therefore$ 1″ : 1·47″ : : 1754 : 7888. Reducing gives $n = 3 \cdot 902$.

Whence the strength is nearly as the fourth power of the side of the square.

I made no experiments to ascertain the inverse power of the length to which the strength is proportional in timber. The great flexibility of such long and slender pieces as would have been necessary to give sufficient variety in their length,—and for the length of the shortest to be a sufficient number of times the thickness, that it would not be sensibly crushed by the breaking weight,—convinced me that it would be as well to seek for this power hypothetically, as to endeavour to obtain it by experiments, which would probably be unsatisfactory.

I therefore assumed (after trying some other hypotheses less successfully,) the strength of timber to be as $\frac{d^4}{l^2}$, which is the simple conclusion of EULER.

Whence $b = \frac{a\,d^4}{l^2}$, where the constant a must be obtained from experiment, for any particular kind of wood.

Dantzic Oak.

The experiments in Table XIII. upon square pillars of this wood flat at the ends, give as below:

1st. From a mean of the results of four experiments, a pillar $60\frac{1}{2}$ inches= 5·04166 feet long, and 1·75 inches square, required 9625 lbs. to break it. Whence, taking the length in feet and the lateral dimensions in inches, we have $\left(\text{in the formula } b = \frac{a\,d^4}{l^2}\right)$ $b = 9625$, $d = 1 \cdot 75$, $l = 5 \cdot 04166$ $\therefore$ $a = \frac{b\,l^2}{d^4} = 26085$.

2nd. From a mean of three experiments upon pillars 46·1 inches $= 3 \cdot 84166$ feet long, and 1·50 inches square, the breaking weight was 7888 lbs.; which gives a, obtained as above, $= 22999$ lbs.

A mean between these two values of a is 24542 lbs.; whence $b = 24542 \frac{d^4}{l^2}$.

Here the length is in feet, the side of the square in inches, and the weight in lbs.; and the rule is only applicable to cases where the pillar is in length so many times the thickness that it is not crushed, as in Articles 6. and 59, by the breaking weight. These remarks apply to the following cases:

Red Deal.

The results from two experiments upon square pillars of this wood, each four feet ten inches long, and two inches square, Table XIII., give $b = 17511 \frac{d^4}{l^2}$; the different values being as above.

LAMANDÉ's *Experiments upon Oak* (*from* TREDGOLD's *Carpentry*).

These experiments were upon square pillars of different lengths, varying as 1, 2, 3, and the sides of their squares as 2, 3, 4. This writer has, in most instances, given four experiments or more to determine the breaking weights of pillars of each size. The longest pillars, compared with their thickness, in this series are 6·375 feet long, and 2·126 inches square, the length being about thirty-five times the side of the square; and the average breaking weight from four experiments was 7769 lbs. We therefore get $a = \frac{b\,l^2}{d^4} = 15455$ lbs., whence $b = 15455\,\frac{d^4}{l^2}$. Further on, I shall endeavour to compute the results of LAMANDÉ's experiments.

Short Pillars of Timber.

65. We have seen that pillars of cast iron, wrought iron, and timber, follow in a great measure the same laws as to their strength (Arts. 5. 58.). In these different materials, pillars with rounded ends have their strength in a constant ratio to that of those with flat ends. This, however, only applies to pillars whose length is so many times the diameter or thickness, that they are not sensibly crushed by the breaking weight.

My experiments have not shown what part of their ultimate crushing weight pillars of timber will bear without suffering such a diminution of their strength, from this cause, as to need correcting for, as was done in estimating the strength of short pillars of cast iron (Art. 41.). In that metal there appeared to be a slight falling off in the strength of even the longest pillars, which I accounted for by the shifting of the position of the neutral line, but it was not such that the results of the general formulæ then used (Art. 36 to 38.) needed modifying; nor was there shown a defect in comparing the breaking weight of the pillars with rounded ends with those with flat ones, till the breaking weight was about one-fourth of the weight which would have crushed the whole pillar without flexure (Art. 6.).

It seems probable, from the experiments in Table XIII., that timber will bear, without much reduction of the breaking weight from crushing, considerably more than one-fourth of the ultimate crushing weight; for the pillars 1·75 inch square and $60\frac{1}{2}$ inches long, with flat ends, exhibited no reduction in comparison of strength with those with rounded ends, though the mean breaking weight of the former was more than one-third of the weight which would have crushed the whole section. It may, however, be remarked, that the circumstance of a pillar with flat ends bearing upwards of three times as much as one of the same diameter with rounded ends, does not prove that the former had not its breaking weight in some degree reduced by crushing, since both descriptions of pillars were considerably compressed. In the absence of other information, therefore, I shall in calculating the strengths of short pillars of timber by the formula $y = \frac{b\,c}{b + c - d}$ (Art. 42.), suppose d, as in cast iron,

to be one-fourth of c, the crushing weight of the section. We have, therefore, as before,

$$y = \frac{b\,c}{b + \frac{3}{4}c},$$

where b = the computed value from the general formulæ for long pillars of different kinds of woods in the last article, and y that for short ones.

If there were no fixed pressure at which pillars in breaking suffered a marked diminution of their strength, but that there was a regular falling off with all weights from the least necessary to break a long pillar up to such as would crush it, then d might be taken = 0, and the formula would be

$$y = \frac{b\,c}{b + c},$$

where b would have to be obtained from experiments upon the longest pillars only, and this formula must be used for all others but them. We see that the strength of short (and perhaps long) pillars to resist fracture by flexure depends upon their resistance to crushing. Before attempting, therefore, to calculate the strengths of the short pillars broken by LAMANDÉ, we must know the crushing strength of the French oak, which fortunately is given by RONDELET*, who states that its strength, from a mean between his experiments, is 6336 lbs. per square inch. Whence we are enabled to obtain an approximation to the value of c, the crushing weight of the pillars which LAMANDÉ made use of.

66. The following table contains the results of his experiments upon short pillars computed by the formula for y, from the value of b, deduced (Art. 64.) from his longest and slenderest pillars, which we may consider not to have been materially reduced in their strength by crushing, and which are placed the first in the Table.

* Traité de l'Art de Bâtir, tome 1, p. 232, edit. 1838.

LAMANDÉ's Experiments upon Square Pillars of Oak.

Length of pillar.	Side of square.	Value of b deduced (Art. 64.) from 1st experiments below.	Value of c computed from RONDELET's experiments.	Breaking weight.		Calculated breaking weight from formula $y = \frac{bc}{b + \frac{3}{4}c}$.
feet.	inches.	lbs.	lbs.	lbs.	Mean.	lbs.
6·375	2·126		28638	7244 7484 8492 7878	7769	
4·25	2·126	17480	28638	11844 12225 13565 12458	12523	12850
2·125	2·126	699205	28638	15631 21296 19993 21060	19495	21908
6·375	3·18	38888	64072	26939 28987 23929 33048 36902	29961	28659
4·25	3·18	87498	64072	43639 36865 36205 28182	36223	41358
2·125	3·18	349993	64072	50958 50958	50958	56207
6·375	4·25	124069	114444	64090 59373 54062 65608	60783	67646
4·25	4·25	279156	114444	100755 85998 73238 96368	89090	87532
2·125	4·25	1116620	114444	95262 105826 94476 88442	96001	106275

The breaking weights above were calculated on a supposition that the oak resisted with a force of 6336 lbs. per square inch. But Mr. BARLOW* states that oak, according to RONDELET (in an early edition of his work), required from 5000 to 6000 lbs. per square inch of base to crush it. If I had adopted these measures of the strength, instead of the former, the calculated values would have been a little lower, and the

* Treatise on the Strength of Timber, &c.

value of d (Art. 65.) might have been assumed somewhat higher. The experimental results of GIRARD from pillars of oak* were calculated, but they agree so little among themselves, that the results of those which were consistent with one another are not given. The reason of the irregularity was attempted to be explained in the early part of this paper (Art. 17.).

Other computations, upon the shortest pillars in Table XIII., agree moderately well with the results of experiment.

Crushing Strength of Timber.

67. To facilitate computations, as to the strength of short pillars of different kinds of wood in the manner above, or otherwise, I shall insert below, from my own experiments, the crushing strength per square inch of some of the descriptions in frequent use, the force being applied in the direction of the fibre.

Description of Wood.	Strength per square inch in lbs.
Alder	6831 to 6960
Ash	8683 to 9363
Baywood	7518 to 7518
Beech	7733 to 9363
American birch	 11663
English birch	3297 to 6402
Cedar	5674 to 5863
Crab	6499 to 7148
Red deal	5748 to 6586
White deal	6781 to 7293
Elder	7451 to 9973
Elm	 10331
Fir (spruce)	6499 to 6819
Hornbeam	4533 to 7289
Mahogany	8198 to 8198
Oak (Quebec)	4231 to 5982
Oak (English)	6484 10058
Oak (Dantzic, very dry)	 7731
Pine (pitch)	6790 to 6790
Pine (yellow, full of turpentine)	5375 to 5445
Pine (red)	5395 to 7518
Poplar	3107 to 5124
Plum (wet)	3654
Plum (dry)	8241 to 10493
Sycamore	7082
Teak	 12101
Larch (fallen two months)	3201 to 5568
Walnut	6063 to 7227
Willow	2898 to 6128

The results in the first column were in each case a mean from about three experiments upon cylinders of wood turned to be one inch diameter, and two inches long, flat at the ends. The wood was moderately dry, being such as is employed in making models for castings. The second column gives the mean strength, as before, from similar specimens, after being turned and kept drying in a warm place two

* Traité Analytique de la Résistance des Solides.

months longer. The lengths of the latter specimens were, in some instances, only one inch, which reduction would increase the strength a little. But the great difference frequently seen in the strength, as given by the two columns, shows strongly the effect of drying upon wood, and the great weakness of wet timber, *it not having half the strength of dry*. The strength of short columns of the same material I have shown to be directly as the area of the section*. This seems easy to conceive, as bodies of the same nature always become crushed by sliding off in an angle, which is nearly constant; the height of the wedge, which would slide off, being in timber usually about half the diameter, or thickness of the specimen.

Relative Strength of Long Pillars of Cast Iron, Wrought Iron, Steel, and Timber.

68. To compare the strengths of pillars, of the same size, in the different materials used in the preceding experiments, I have computed, by the rules used for cast iron (Art. 36–38.), the strength of a pillar, of the same diameter and length, in each body, taking the results from the longest pillars, in order that they may not be crushed (as described in Art. 6.) by the breaking weight. The relative strengths are as below:

Calling the strength of cast iron 1000, we have for wrought iron 1745, cast steel 2518, Dantzic oak 108·8, red deal 78·5. These numbers, except the last, are all computed from the pillars with rounded ends.

* Transactions of the British Association, vol. vi.

Results of Experiments on the Resistance of Solid Uniform Cylinders of Cast Iron to a Crushing Force.

Table I.—Low Moor Iron, No. 3. cast in dry sand. Plate XV. No. 1, A. Ends of Specimens turned, so that the force would pass through the axis.

Length.	Diameter.	Mean Diameter.	Deflection of middle of pillar.	Corresponding weight.	Breaking weight.	Mean from breaking weights.	Remarks.
inches.	inch.	inch.	inch.	lbs.	lbs.	lbs.	
60·5	·50	....	·07	58			
		·50	·24	95			These pillars, and those of the same length below, were made by mistake half an inch longer than was intended; and therefore all the future ones were made of the same length, or exact sub-divisions of it.
			·49	113	136		
60·5	·50	....	·04	97		143	
			·08	113			
			·23	136	150		
60·5	·77	·77	....	....	780	780	
60·5	·77				780		
60·5	·99		·05	515			
			·10	655			
			·14	991			
		·99	·19	1183			
			·32	1471			
			·52	1615	1663	1902	Broke 2 inches from middle.
60·5	·99	....	....	....	2141		Broke 1¼ inch from middle.
60·5	1·28	1·29	·25	5069	5293		
60·5	1·30		·07	5673		5707	
			·10 ?	5897	6121		Time of making the experiment 1¼ hour.
60·5	1·29		·03	2141			Broke 2¾ inches from middle.
			·06	3653			The two following pillars were cast in green sand.
			·17	4549			
		1·295	·34	4997	5149		Weight of pillar.
			·00 ?	2141			19 lbs. 8 oz.
60·5	1·30		bent.	2757			
			·005	3653			
			·03	4549		5465*	Mean 19 lbs. 11 oz.
			·07	5445			
					5781		19 lbs. 14 oz.
60·5	1·53	1·52	·14	10525	10861	10861	T : C : : 126 : 26.
60·5	1·51				10861		Weight of pillar.
60·5	1·53	1·535			10121	10650*	28 lbs. 5 oz.
60·5	1·54				11179		28 lbs. 9 oz. Mean 28 lbs. 7 oz.
							Cast in green sand.
60·5	1·76		·01 ?	8483			
			·09	11035			
		1·765	·20	13587			
			·30	14225	14701	15560	
60·5	1·77	....	....	....	16419		
60·5	1·76		bent.	2808			
			·02	8617			
		1·78	·07	12445			
			·12	13721			Weight of pillar 38 lbs.
			·22	15233	16493		
60·5	1·80		bent.	3355			
			·05	11649			
			·09	14201		17564*	
			·18	16115			
			·30	17795			
			·48	18355	18635		T : C : : 145 : 36.

Table I. (Continued.)

Length.	Diameter.	Mean Diameter.	Deflection of middle of pillar.	Corresponding weight.	Breaking weight.	Mean from breaking weights.	Remarks.
inches.	inch.	inch.	inch.	lbs.	lbs.	lbs.	
60·5	1·94	1·94	....	....	22811	22811	Weight of pillar 44 lbs.
60·5	1·97		bent.	3355			
			·02	7386			
			·07	12970			
			·20	19943			
			·23	21035			
		1·96	·28	22127			
			·37	23219			
			·50	24311			
			·52				
			·60	24857	25403		
60·5	1·96		·03	12287			
			·05	14839			Weight of pillar.
			·11	17391			{47 lbs.
			·19	19943		24291*	46 lbs. 8 oz.} Mean 46 lbs. 12 oz.
			·20	21219			
			·35	22395			
			·48	22787	23179		T : C : : 150 : 42, or 153 : 43.
30·25	·50		bent.	248			
			·03	360			
		·50	·15	472	526	......	Broke 2⅞ inches from middle.
30·25	·50		·02	304		539	
			·09	472	535		Broke near the middle.
30·25	·50		....	....	556	......	Broke ¾ inch from middle.
30·25	·77	....	·02	1717			
			·07	2054			
		·77	·10	2390	2726	2726	For form of fracture, see *f*. Plate XV.
30·25	·99		·04	2745			
		·99	·13	4985	6105	......	T : C : : 76 : 23.
30·25	·99		·02	3641			
						6105	T : C : : 78 : 21.
			·07	4985	6105	......	This bent apparently in different directions. For the forms of fracture, see *c d e*. Plate XV.
30·25	1·29		·01	12287	....	....	Neutral line well-defined.
			·03	13563			
		1·29	·04	14839			
			·07	16115	17515	......	T : C : : 96 : 34; in another case 95 : 32.
30·25	1·29		·08	12287			
			·10	13563		17235	
			·12	14839			T : C :: 105 : 24 ?, small flaw in tensile part.
			·21	16115	16955	......	For the forms of fracture, see *a b*. Plate XV.
30·25	1·52	....	·04	22619			
			·06	25867	....	....	T : C :: 99 : 51.
			·12	30739	32419		
30·25	1·53	1·52	....	....	34638	32531	T : C : : 101 : 52.
30·25	1·51	....	·07	22619	30536		T : C : : 110 : 43.
20·1666	1·00	1·01	....	....	15737	15737*	Weight 4 lbs. 3 oz.; T : C : : 63 : 67.
20·1666	1·02		....	....	15737		Weight 4 lbs. 5 oz.; T : C : : 65 : 37. For form of fracture, see *h*. Plate XV.

TABLE I. (Continued.)

Length.	Diameter.	Mean Diameter.	Deflection of middle of pillar.	Corresponding weight.	Breaking weight.	Mean from breaking weights.	Remarks.
inches.	inch.	inch.	inch.	lbs.	lbs.	lbs.	
20·1666	·785	·767	....	....	7255	6602*	Weight 2 lbs. 8½ oz.
20·1666	·75		....	....	5950		Weight 2 lbs. 6 oz.
15·125	·50	....	·05	1689			
			·09	1801			
			·11	1913			Neutral line well defined.
			·31	1997	1997		T : C : : 39 : 11.
15·125	·50	....	·20	1801			T : C : : 39 : 11.
			·34	1857	1857		In this experiment the deflection was taken as the pillar was bending.
15·125	·50	....	·08	1353			
			·15	1689			
		·50	·20	1801	1857	1904	
15·125	·77	....	....	....	10138	....	T : C : : 48 : 29.
15·125	·76	....	....	....	9746	....	T : C : : 53 : 23.
15·125	·75	·76	....	....	7786	9223	T : C : : 49 : 26. For the forms of fracture, see *g*. Plate XV.
15·125	·99	....	....	....	20163	....	T : C : : 57 : 42.
15·125	·99	....	....	....	19239	....	T : C : : 60 : 39.
15·125	·99	·99	....	....	19855	19752	T : C : : 60 : 39. For forms of fracture, see *i*, *j*, *k*. Plate XV.
10·083	·76		....	....	16683		The two first failed by the ends becoming split, by a conical wedge forming at them.
10·083	·76	·76	....	....	16683	17506	
10·083	·77		....	....	19152		
7·5625	·51	....	....	....	6188	....	T : C : : 31 : 20.
7·5625	·49	....	....	....	4578	....	T : C : : 32 : 17.
7·5625	·49	·497	....	....	5019	5262	T : C : : 34 : 15.
7·5625	·77	·77	....	....	23893	....	T : C : : 35 : 42.
7·5625	·77		....	....	22003	22948	T : C : : 41 : 36. These pillars were split at both ends. For fractures, see *l*, *m*. Plate XV.
3·7812	·50	....	....	....	15233	15107	In these two experiments the area compressed seemed to have been greater than the extended area. The ends were split by the pressure. For form of fracture, see *n*. Plate XV.
3.7812	·50	·50	....	....	14981		

By the letters T, C are to be understood the versed sines or depths of the neutral line on the surfaces submitted to tension or compression; and the ratio T : C is that of those versed sines or depths. The neutral line is not, strictly speaking, a straight one, but a curve; the dimensions are, however, so adapted that it may be considered as a straight line.

The pillars marked with an asterisk were cast after the others, and from the same description of iron, but probably a different sample; some of them were cast in green (moist) sand, too, whilst all the rest of the pillars were cast in dry sand.

In all cases, except otherwise mentioned, the pillars broke nearly in the middle, both in this Table and the following one.

Results of Experiments on the Resistance of Solid Uniform Cylinders of Cast Iron to a Crushing Force.

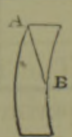
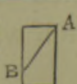
TABLE II.—Low Moor Iron, No. 3, cast in green sand, Plate XV. No. 1, C. Ends of cylinders turned *flat* and parallel to each other, and the pressure caused by the approach of two parallel surfaces, between which the cylinder was placed, its ends perfectly coinciding with them.

Length.	Diameter.	Mean Diameter.	Weight.	Deflection of middle of pillar.	Corresponding weight.	Breaking weight.	Mean from breaking weights.	Remarks.
inches.	inch.	inch.	lbs. oz.	inch.	lbs.	lbs.	lbs.	
60·5	·51	·51	3 4¼	......	....	483	487	These two had discs two inches diameter upon the ends; all the rest had the ends turned flat, and were without discs.
60·5	·51		3 5¼			491		
60·5	·77	·77		·07	1162			
				·11	1812			
				·22	2036	....	....	After bearing 2036 it bent in another direction.
				·30	2260	2316		
60·5	·77		7 5½	·06	1588			
				·08	1812		2456	
				·10	2036			
				·13	2260			
				·17	2484	2596		
60·5	·99			·05	4123			
				·12	5467			
				·21	6139			
				·30	6475	6811		
60·5	1·01	·997		·05	4123			
				·14	5467	5971	6238	
60·5	·99	....	11 8	·04	4123			
				·10	5467	5932		
60·5	1·30	....	....	·10	11235			
				·11	11627			
				·14	12411			
				·16	13195			
				·19	13979			
				·24	14763			
				·30	15547			
				·47	16331	16527		
60·5	1·29	1·29	19 11	·15	11217			
				·22	13177			
				·26	13961		16064	
				·37	14745			
				·45	15137	15333		
60·5	1·28	....	20 0	....	....	16331		

TABLE II. (Continued.)

Length.	Diameter.	Mean Diameter.	Weight.	Deflection of middle of pillar.	Corresponding weight.	Breaking weight.	Mean from breaking weights.	Remarks.
inches.	inch.	inch.	lbs. oz.	inch.	lbs.	lbs.	lbs.	
60·5	1·55		28 10	·13	21857			
				·15	23481			
				·21	25105			
				·48	26729			
				·62	27135	27135	....	Broke immediately after the deflection was taken.
60·5	1·57		29 7	·10	21857			
				·12	23481			
		1·56	....	·14	25105			
				·16	26729			
				·20	28353		28962	
				·26	29977			
				·33	30789			
				·48	31398	31398	....	T : C :: 120 : 37. Broke in about five minutes. A wedge broke out, showing the neutral line.
60·5	1·55		28 5	....	....	28353		
30·25	·50	·50	1 10½	·05	1090			
				·09	1494			
				·12	1550			
				·13	1606			
						1662	1662	
30·25	·78			·08	4357			
				·10	5701			
				·13	7045			
				·22	7717	8389		
30·25	·78			·04	2905			
				·05	3801			
		·77		·06	4697			
				·09	7833		8811	
				·10	8281			
				·13	8729			
				·19	9177	9625		
30·25	·76			....	....	8420	....	T : C :: 61 : 15. A crack showed the boundaries of tension and compression.
30·25	1·01			....	....	19132		
30·25	1·00			·05	13833			
				·08	15849			
				·10	16857			
				·19	17865	18369		
30·25	1·02			·05	17795			
				·06	18915			
				·10	20035			
		1·01		·13	21155		20310	
				·16	21715	21844	....	Cracked neutral line. T : C :: 64 : 39.
30·25	1·00			·05	14841			
				·06	15849			
				·07	16857			
				·07	17865			
				·08	18873			
				·10	19881			
				·13	20889	21897	....	T : C :: 65 : 35.

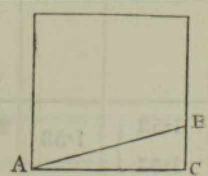
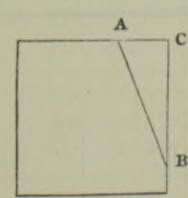
Table II. (Continued.)

Length.	Diameter.	Mean diameter.	Weight.	Mean weight.	Breaking weight.	Mean breaking weight.	Ratio T : C.	Remarks.
inches.	inch.	inch.	lbs. oz.	lbs. oz.	lbs.	lbs.		
20·1666	·51	·51	1 1½	1 1⅜	3830	3830		
20·1666	·51		1 1¼		3830			
20·1666	·78	·777	2 9	2 8⅓	16701	15581	45 : 33	The first broke in two pieces near to the middle, and near to one end; and a piece was split off the other end, at the neutral line. The other two broke nearly in the same manner. For figure of fracture, see *q*, Plate XV.
20·1666	·78		2 8½		15357			
20·1666	·77		2 7½		14685			
20·1666	1·03	1·022	4 6	4 5½	32007	31804	56 : 46	Broke in two places near middle; both ends cracked at neutral line. For figure of fracture, see *o*, *p*, Plate XV.
20·1666	1·015		4 5		31601		54 : 49	
15·125	·51	·51	14	13$\frac{10}{12}$	6512	6764	32 : 19	
15·125	·51		13¾		7016			
15·125	·51		13¾		6764			
15·125	·79	·775	29¾	29$\frac{3}{16}$	21179	21059		
15·125	·77		29		22363			
15·125	·76		28½		19003			
15·125	·78		29½		21691			
15·125	1·00	1·00	3 4	3 4	39112	40250	. . . . .	For form of fracture, see *r*, *s*, Plate XV.
15·125	1·00		3 4		41388			
12·1	·50	·50	10½	10⅙	7279	7195	3 : 2	
12·1	·50		10		6943		3 : 2	
12·1	·50		10		7363		3 : 2	
12·1	·78	·785	1 7½	1 7¾	25355	24287	39 : 39	
12·1	·79		1 8		24043		1 : 1	
12·1	·79		1 8¼		23875		1 : 1	
12·1	·78		1 7½		23875			
10·083	·50	·50	8½	8½	8287	8931	13 : 12	
10·083	·50		8½		8623		13 : 12	
10·083	·50		8½		9883		13 : 13	
10·0833	·78	·768			27491	25923	1 : 1	These generally broke in several pieces; but always in the middle by bending. There was, however, usually a wedge formed about the centre which tended to split the pillar there.
10·0833	·77				25531			
10·0833	·76				25531			
10·0833	·76				25139			
7·5625	·50	·50	6⅓	6⅓	11479	11255	12 : 13	
7·5625	·50		6⅓		11143		12 : 13	
7·5625	·50		6⅓		11143		12 : 13	
7·5625	·78	·777	14½	14$\frac{5}{12}$	33225	32007	4 : 5	There was a good deal of doubt respecting the neutral line; but somewhat more than one-half was compressed. For fracture, see *t*, *u*, *v*, Plate XV.
7·5625	·78		14½		31601			
7·5625	·77		14¼		31195			
3·7812	·50	·50			17795	17468	20 : 30	These broke in the middle by bending as before, but they generally showed a short ridge or wedge in the centre, as mentioned above.
3·7812	·50				17935			
3·7812	·50				16675			
2	·52	·52			23035	22867		The first bent and slid off in A B. The other bent and cracked half across in the middle. 
2	·52				22699			
1	·52	·52			23963	24616		Broke by wedge, about three quarters of an inch high, sliding off in the direction A B. 
1	·52				24747			
1	·52				25139			

Further remarks on Table II. The ratio T : C shows the position of the neutral line, as described in the notes to Table I. The two pillars that stand first in this Table had discs upon their ends, two inches diameter, turned flat; and the weight of the pillar, as set down, is that without the discs. These were added on account of the smallness of the diameter, ·51 inch, as compared with the length, $60\frac{1}{2}$ inches, to give the ends a firmer bearing; but in all other instances, the pillars in this Table were without discs.

The forms of fracture *o*, *p*, *q*, &c. Plate XV., referred to in these Tables, and the marks upon the pillars over them, show the manner, and into what number of pieces, some of the specimens broke, though the figures are not drawn to a scale. The exact dimensions of these pillars will be obtained from the Tables.

Table III.—Rectangular, or Square Uniform Pillars of Low Moor Iron, No. 3, $60\frac{1}{2}$ inches long, rounded at the ends. Plate XV. No. 3, A, B, C.

Dimensions of section.	Mean dimensions, or side of equal square.	Weight of pillar.	Mean weight.	Deflection.	Weight causing the deflection.	Breaking weight.	Mean breaking weight.	Remarks.
inch. inch.	inch.	lbs. oz.	lbs. oz.	inch.	lbs.	lbs.	lbs.	
·77 × ·77	·7716	9 $3\frac{2}{5}$	9 $3\frac{2}{5}$			1251	1269	Broke by bending in direction of diagonal.
·77 × ·78		9 $3\frac{2}{5}$				1307		Broke by bending in direction of diagonal.
·77 × ·77		9 $3\frac{2}{5}$				1251		
1·02 × 1·04	1·0275	16 7	16 $3\frac{1}{2}$	·01 ?	1118		3968	Broke three inches from middle, by bending in direction of diagonal, a wedge ·25 inch deep separating from corner, the diagonal being 1·45 inches long.
				·02	1364			
				·04	2036			
				·08	2708			
				·16	3380			
				·23	3716			
				·43	4052	4164		
1·03 × 1·02		16 0		·01 ?	1118			Broke as before, depth of wedge ·28 inch.
				·02	1364			
				·05	2036			
				·09	2708			
				·22	3380			
				·45	3716	3772		
1·54 × 1·52	1·540	35 9	35 $15\frac{1}{2}$	bent	3355		16127	A B, the neutral line, the triangle being compressed. B C = ·35 inch.  After bearing 14203 lbs. it was unloaded and returned to ·01 inch.
				·01	5467			
				·02	7483			
				·05	10843			
				·17	14203			
				·33	15861	16673		
1·54 × 1·56		36 6		bent	2141			A B, the neutral line. A C, = ·45 inch. 
				·015	4465			
				·02	6481			
				·11	11169			
				·23	13565			
				·35	14461			
				·44	14909			
				·52	15357	15581		

The Table above may be considered as a supplement to Table I., the object of the experiments in this being, to ascertain whether square pillars of different diameters varied in their strength according to the same laws that round ones do, a matter which will be examined into further on.

Table IV.—Strength of Solid Uniform Cylindrical Pillars of Low Moor Iron, No. 3, $60\frac{1}{2}$ inches long, with discs upon their ends to prevent them turning. Plate XV. No. 2, fig. 3.

Diameter of pillar.	Mean diameter.	Weight of pillar.	Deflection.	Corresponding weight.	Breaking weight.	Mean breaking weight.	Remarks.
inch.	inch.	lbs. oz.	inch.	lbs.	lbs.	lbs.	
·51	·51				483	487	
·51					491		
·78		8 9	·16	1089	....	......	After bearing the first weight it had taken a set of about ·01. Broke $4\frac{1}{8}$ inches from middle, at a slight flaw.
			·23	1617			
			·26	1841			
	·775		·31	2065			
			·35	2289			
			·44	2513			
			·61	2737	2926	2719	
·77		7 10			2513		Broke $1\frac{7}{8}$ inch from middle.
1·00		12 12	·06	4123			
			·15	5467			
			·34	6811			
			·48	7147			
				7315	7483		Broke in middle.
·99	1·00	12 9	·04	4095			
			·06	5439			
			·10	6111		6830	
			·21	6783	7117		Broke $\frac{3}{8}$ inch from middle, and the disc at one end was split.
1·00		12 10			6907		
1·00		12 9			5815		
1·28		21 5	·01 ?	2381			
			·05	4973	....	......	After bearing 4973 it was unloaded and returned to ·00.
			·11	11169			
			·22	14405			
	1·28		·30	15301			
			·36	15749			
			·51	16197	16421		Broke 2 inches from middle and near one end. T : C : : 101 : 28.
1·28		21 8	·01 ?	2749			
			·02	5437			
			·18	13797		16369	After 5437 it returned to ·00.
			·25	14805			
			·38	15813			
			·62	16317	16317	......	Broke in about a minute, $1\frac{5}{8}$ inch from middle, and near one end. A wedge showed T : C : : 91 : 36.
1·53	1·53	29 4			32007	30789	Broke in middle T : C : : 116 : 37.
1·53					29571		This was a little bent, and probably weakened by making straight.

Table V.—Uniform Pillars of Low Moor Iron, No. 3, with one end rounded and one flat, placed between others, with both ends rounded and both flat.

Description.	Length of pillar.	Diameter.	Mean diameter.	Weight of pillars.	Deflection.	Weight causing the deflection.	Breaking weight.	Mean breaking weight.	Remarks.
	inches.	inch.	inch.	lbs. oz.	inches.	lbs.	lbs.	lbs.	
Both ends rounded. Plate No. 1, A.	30¼	·79	·78	3 13	....	....	2953	3017	
Both ends rounded. No. 1, A.	30¼	·78		3 12	....	....	3372		
Both ends rounded. No. 1, A.	30¼	·77		....	....	....	2726		
One end rounded and one flat. No. 1, B.	30¼	·78	·78	3 12	....	....	6278	6278	
Both ends flat. No. 1, C.	30¼	·78	·78	....	....	....	9625	9007	
Both ends flat. No. 1, C.	30¼	·78		....	....	....	8389		
Both ends rounded. No. 1, A.	30¼	1·00	1·00	6 2	....	....	7397	7009	
Both ends rounded. No. 1, A.	30¼	1·01		6 2⅓	....	....	7769		
Both ends rounded. No. 1, A.	30¼	1·01		6 4½	....	....	7672		
Both ends rounded. No. 1, A.	30¼	·99		....	....	....	6105		
Both ends rounded. No. 1, A.	30¼	·99		....	....	....	6105		
One end rounded and one flat. No. 1, B.	30¼	1·02	1·00	6 4	....	....	14685	13499	
One end rounded and one flat. No. 1, B.	30¼	·98		5 14	....	....	12314		
One end rounded and the other flat with a disc of double the diameter. No. 2, B.	30¼	1·00	1·00	6 7	·09	12445	....	....	Broke 10·42 inches from the rounded end, after bearing the weight some minutes.
					1·30	12893			
					2·95	13789	13789	13565	
One end rounded and the other flat with a disc of double the diameter. No. 2, B.	30¼	1·00		6 7	·13	12445			Broke 10·30 inches from rounded end.
					·20	12893	13341		
Both ends flat. No. 1, C.	30¼	1·01	1·01		....	....	19132	20310	
Both ends flat. No. 1, C.	30¼	1·02			....	....	21844		
Both ends flat. No. 1, C.	30¼	1·00			....	....	18369		
Both ends flat. No. 1, C.	30¼	1·00			....	....	21897		
Both ends flat with discs. No. 2, C.	30¼	1·00	1·00	6 15	·05	12445			
					·06	14685			
					·07	16925			
					·09	18045			
					·15	20285			
					·27	21629	21853	22475	
Both ends flat with discs. No. 2, C.	30¼	1·00		6 15	·02	14685			
					·04	16925			
					·06	19165			
					·09	21405			
					1·50	22525	23097		
Both ends rounded. No. 1, A.	60½	1·76	1·76	38 0	....	....	16493	16493	
One end rounded and one flat. No. 1, B.	60½	1·74	1·74	37 0	....	....	33557	33557	

TABLE VI.—Pillars enlarged in the middle: ends rounded.

Experiments to ascertain the Resistance of Pillars of Cast Iron, the Low Moor, No. 3, formed in the shape of frustums of double cones, the bases meeting in the middle, and the extreme ends left prominent in the centre, that the force might pass through the axis. Length of pillar $60\frac{1}{2}$ inches. Plate XV. No. 4, A, B, C, D.

End diameter.	Middle diameter.	Weight of column.	Breaking weight.	Mean breaking weight.	Strength, or breaking weight of a uniform cylindrical pillar of the same weight as the mean.	Gain in parts of mean breaking weight.	Remarks.
inch.	inch.	lbs. oz.	lbs.	lbs.	lbs.		
1·01	1·27	16 5	4521	4521	3838	$\frac{1}{6 \cdot 62}$	This pillar broke five inches from the middle, where the diameter was 1·24.
1·05	1·56	20 12 } 21 lbs. 1 oz.	6453	} 7085	6205	$\frac{1}{8 \cdot 05}$	The last of these broke ten inches from the middle, where the diameter was 1·43; and also $11\frac{1}{4}$ inches from the middle at the other end.
1·05	1·55	21 0	7573				
1·03	1·56	21 6	7229				
1·04	1·85	26 8	11069	11069	9555	$\frac{1}{7 \cdot 31}$	This broke fifteen inches from the middle and toward the other end.
1·05	1·82	....	10985	10985	....	....	This broke $16\frac{3}{4}$ inches from the end, at a flaw, it is therefore not used for comparison.
1·00	1·97	28 0	12245	12245	10597 } 10344 } Mean 10470.	$\frac{1}{6 \cdot 89}$	This broke 20·35 inches from the middle, where the depth of the tensile part was to that of the compressed as 105 : 27.

The strengths of the uniform pillars, of equal weight with those of variable thickness, in the Table above, were calculated from the experiments upon the pillars whose mean weights are 19 lbs. 11 oz., and 28 lbs. 7 oz., in Table I.: it having been shown (Art. 46.) that the strength, where the length is the same, varies as the 1·88 power of the weight, in pillars with rounded ends.

Table VII.—Pillars enlarged in the middle, differing from the last only in having discs upon the ends, two inches diameter, and half an inch thick, turned flat. Length $60\frac{1}{2}$ inches. Plate XV. No. 4, E, F.

End diameter.	Middle diameter.	Weight of pillar.	Mean weight of pillar.	Deflection.	Corresponding weight.	Breaking weight.	Mean breaking weight.	Breaking weight of uniform pillars of same weight, with discs.	Remarks.
inch.	inch.	lbs. oz.	lbs. oz.	inch.	lbs.	lbs.	lbs.	lbs.	
1·04	1·31	16 10		·03	4251				The first of these pillars, with enlarged middle, broke $9\frac{1}{2}$ inches from the middle, and had both discs split. A wedge breaking out, at the place of fracture, showed the depth of tension to that of compression as 104 : 16.
				·05	8171				The next pillar gave the ratio 104 : 22, breaking $7\frac{1}{4}$ inches from the middle, as well as one end breaking off, and the disc at the other splitting.
				·09	8955				
				·12	9739				
				·17	10525				
				·28	11307	12091			
1·00	1·27	16 4	16 10			10915	11526	11756	The third pillar broke 11·6 inches from the middle, and had both discs split. The pillar, with enlarged middle, bore $\frac{1}{50}$th less than the uniform pillar.
1·03	1·27	17 1				11573			
1·06	1·56	21 13	21 13	·06	13699				This pillar, with enlarged middle, broke $7\frac{3}{4}$ inches from middle; it bore $\frac{1}{8·56}$th part more than the uniform one.
				·07	14459				
				·12	15803				
				·16	17147				
				·29	18491				
				·50	18939	19163	19163	16924	

The strengths of the uniform disced pillars, of the same weight and length as those with enlarged middles, were calculated according to the assumption that the strength varied as the 1·775 power of the weight. See Art. 46.

The first was obtained from a mean between the results of two experiments, where an uniform disced pillar of the same length, and 15 lbs. 14 oz. weight, was broken with 10831 lbs.; and the second, from a like mean, in which a disced pillar of that length, and 21 lbs. $6\frac{1}{2}$ oz. weight was broken with 16369 lbs.

Table VIII.—Hollow Cylinders rounded at the Ends.

Results of Experiments on the Strength of Hollow Cylinders of Cast Iron (the Low Moor, No. 3.), the ends having hemispherical caps upon them, that the compressing force might act through the axis of the pillar. Length of cylinder, including caps on the ends, 7 feet 6¾ inches. Plate XV. No. 7, A, B, C, &c.

No. of Experiment.	Description of Pillar.	Deflection.	Weight producing the deflection.	Breaking weight, or that with which it sunk.	Value of x from formula $x = \frac{W}{D^{3.76} - d^{3.76}}$, where W = the breaking weight, and D, d the external and internal diameters.	Remarks.
	inches.	inch.	lbs.	lbs.	lbs.	
1.	Hollow uniform cylinder. External diameter ...... 1·78 Internal diam. by measure 1·21 Internal diameter by calculation .... } 1·21 Mean... 1·21 Weight of cylinder, 7 ft. 6¾ inches long, 31 lbs.	·03 ·07 ·11 ·16 ·20 ·32 ·49	2237 2813 3317 3821 4325 4829 5333	5585	834·37	With the weight 5585 lbs. the pillar sunk down, but was not allowed to bend so as to break. It was preserved in order that an experiment might be made upon it in another way. When unloaded it became nearly straight, and was easily rendered so. The same plan was adopted with several other pillars, as it was found that, by using care, this experiment might be made with little injury to the pillar. The weight of the pillars, as set down, and the values of x, as calculated, are for a length of 7 ft. 6¾ inches, though the pillars were two inches shorter.
2.	Hollow uniform cylinder. External diameter ...... 1·74 Internal diam. by measure 1·20 Internal diameter by calculation .... } 1·174 Mean... 1·87 Weight of cylinder, 7 ft. 6¾ inches long, 30¼ lbs.	·02 ·03 ·05 ·07 ·13 ·20 ·36 ·48	2141 2813 3317 3821 4325 4829 5333 5585	5711	933·13	This pillar sunk with the weight 5711 lbs.; but was prevented from breaking, as before, in order that another experiment might be made upon it. When unloaded it had taken a permanent set of ·07 inch; but it had been found by the previous experiments that pillars slightly bent might be rendered straight, and used again with little damage, as was done with the preceding one.
3.	Hollow uniform cylinder. External diameter ...... 2·01 Internal diam. by measure 1·40 Internal diameter by calculation .... } 1·43 Mean... 1·415 Weight of cylinder, 7 ft. 6¾ inches long, 36½ lbs.	·04 ·13 ·21 ·31 ·58 ·75	2237 4829 5837 6845 7853 8105	8357	826·20	It sunk down as usual, and when unloaded had taken a permanent set of one-tenth of an inch. Thickness of metal at place of fracture varied on the opposite sides as 19 : 42. This pillar was not reduced near the ends.
4.	Hollow uniform cylinder. External diameter ...... 2·33 Internal diam. by measure 1·70 Internal diameter by calculation .... } 1·70 Mean... 1·70 Weight of pillar, length as above, 46½ lbs.	·24 ·30 ·37 ·46 ·61 ·72	11169 11953 12737 13521 14305 14697	15089	903·28	It sunk down as usual, but was prevented from bending so as to break: it was broken afterwards. The core did not pass through the centre: the thicknesses of the opposite sides at the place of fracture were as 1 : 4 nearly; the thinner side having been the compressed one, as was often the case.
5.	Hollow uniform cylinder. External diameter ...... 2·23 Internal diam. by measure 1·53 Internal diameter by calculation .... } 1·55 Mean... 1·54 Weight of pillar 47 lbs., length as above.	·01 ·06 ·12 ·22 ·28 ·37 ·45 ·55 ·69	2237 4325 6341 8357 9365 10373 10877 11381 12137	12389	808·21	It sunk as usual, but was prevented from breaking. This pillar was not reduced at the ends.
6.	Hollow uniform cylinder. External diameter ...... 2·24 Internal diam. by measure 1·71 Internal diameter by calculation .... } 1·76 Mean... 1·735 Weight of pillar 34¾ lbs.	·015 ·02 direction altered. ·04 ·34 ·38	2141 7101 8165 12445 12669	13341	1041·7	

Table VIII. (Continued.)

No. of Experiment.	Description of Pillar.	Deflection.	Weight producing the deflection.	Breaking weight, or that with which it sunk.	Value of x from formula $x = \frac{W}{D^{3.76} - d^{3.76}}$, where W = the breaking weight, and D, d the external and internal diameters.	Remarks.
	inches.	inch.	lbs.	lbs.	lbs.	
7.	Hollow uniform cylinder. External diameter ...... 2·24 Internal diam. by measure 1·58	·01 ? ·02 ·05 ·09 ·17 ·21	2141 4325 6341 11169 13129 13521	13913	917·62	After bearing 6341 lbs. the pillar was unloaded and returned to ·00 deflection. This pillar was reduced to half its thickness near to the ends, and to three-fourths half way between the middle and each end; but it did not fail through this, and sunk by flexure, the curvature being greatest in the middle, as in other cases.
8.	Hollow uniform cylinder. External diameter ...... 2·49 Internal diam. by measure 1·89 Internal diameter by calculation .... } 1·89 Mean... 1·89 Weight of pillar 48¾ lbs.	bent. ·01 ? ·02 ·04 ·00 ·19 ·40 ·52	3355 4123 6139 10171 unloaded. 16115 18623 19239	19855	996·24	This pillar was reduced in the same way as the last. When broken, which took place eight inches from the middle and not at the reduced parts, it showed that the ver sin, or depth of tension, was to that of compression as 19 : 6 nearly. The casting was very good, and the core in the middle.
9.	Hollow uniform cylinder. External diameter ...... 2·47 Internal diam. by measure 1·99 Internal diameter by calculation........... ...... } 1·97 Mean... 1·98 Weight of pillar 41 lbs.	·01 ? ·02 ·23 ·32 ·62	3211 8553 17391 18029 18667	19003	1123·5	Variation of thickness of metal at place of fracture 7 : 9.
10.	Hollow uniform cylinder. External diameter ...... 2·46 Internal diameter by measure .... } 1·87 Internal diameter by calculation .... } 1·84 Mean... 1·855 Weight of pillar 49 lbs.	bent. ·01 ? ·02 direction altered. ·04 ·09 ·14 ·28 ·40 ·49 ·65	2141 4581 7241 9103 12445 14359 16487 17551 18083 18615	19147	989·95	It broke at a part rather unsound, 13½ inches from the middle. The thicknesses of the metal on opposite sides, at the place of fracture, were as 3 : 4.
11.	Hollow uniform cylinder. External diameter ...... 2·73 Internal diam. by measure 2·14 Internal diameter by calculation .... } 2·20 Mean... 2·17 Weight of pillar 48 lbs.	bent. ·01 ? direction altered. ·03 ·05 ·40 ·70	3603 8185 12105 16025 21219 22787	23963	949·48	When broken, the cylinder was somewhat thinner on one side than the other, and the thin side was that which was compressed; as was the case in most of the instances where the cylinders were not equal in thickness at both ends.
12.	Hollow uniform cylinder. External diameter ...... 2·74 Internal diameter by measure .... } 2·14 Internal diameter by calculation .... } 2·17 Mean... 2·155 Weight of pillar 51¼ lbs.	·02 ·05 ·07 ·08 ·14 ·37 ·62 1·10	3603 8185 12105 16025 21219 25139 26707 27491	27883	1059·5	This broke three inches from the middle, and a crack showing the neutral line gave the ver sin of the part submitted to tension = 1·76 inches. Variation of thickness of metal at place of fracture 2 : 3 nearly.
13.	Hollow uniform cylinder. External diameter ...... 3·01 Internal diam. by measure 2·45 Internal diameter by calculation .... } 2·51 Mean... 2·48 Weight of pillar 50¼ lbs.	·07 ·14 ·23 ·30 ·44 ·75	16115 18667 21219 22787 24355 25923	26707	819·46	This casting was allowed to break, and the thicknesses of the metal at opposite sides of the place of fracture were as 9 : 15. The thin side was the compressed one. The depth or ver sin of compression was to that of extension as 1 : 5.

Table VIII. (Continued.)

No. of Experiment.	Description of Pillar.	Deflection.	Weight producing the deflection.	Breaking weight, or that with which it sunk.	Value of x from formula $x = \frac{W}{D^{3\cdot76} - d^{3\cdot76}}$, where W = the breaking weight, and D, d the external and internal diameter.	Remarks.
	inches.	inch.	lbs.	lbs.	lbs.	
14.	Hollow uniform cylinder. External diameter ...... 3·36 Internal diameter by measure ...... 2·81 Internal diameter by calculation ...... 2·836 Mean... 2·823 Weight of pillar, 7 ft. 6¾ inches long, 59½ lbs.	·09 ·11 ·13 ·19 ·23 ·27 ·32 ·41 ·50 ·61 ·69 ·81 1·10	16115 18667 21219 23571 25923 28275 30627 32979 35331 36507 37783 39057 40335	 40973	.... 895·02	The thicknesses of the metal, on opposite sides, at the place of fracture, were as 19 : 34; the thinner side being the compressed one.
15.	Hollow uniform cylinder. External diameter ...... 3·36 Internal diameter by measure ...... 2·61 Internal diameter by calculation ...... 2·65 Mean... 2·63 Weight of pillar, 7 ft. 6¾ inches long, 77¾ lbs.	bent. ·09 ·13 ·15 ·17 ·24 ·30 ·38 ·48 ·59 ·67 ·87 ·90 1·07	3355 16115 18667 21729 24148 28986 33824 37701 41632 43597 45563 47528 48511 49494	 50477	.... 880·11	The thickness of the metal varied as 30 : 43, or 5 : 7 nearly, the thinner side being compressed.
16.	*Solid* uniform pillar cast in green sand. Diameter 2·24 inch. Weight 93 lbs. It was rounded at the ends.	bent. ·005 ? ·02 ·00 ·05 ·12 ·25	2141 5161 7541 unloaded. 15737 18761 20273	 21281	.... 1026·6	This experiment, on a solid cylindrical pillar, is given for comparison with the others, which were all on hollow ones.
17.	Pillars of shorter lengths than those above. Hollow uniform cylinder. External diameter 1·78 inch. Internal diameter 1·21 inch. Length 4 ft. 9 inches.	....	....	 13693	.... 927·86	In this casting the metal was nearly of equal thickness all round. The value of x or 927·86, is reduced by calculation to that of a pillar 7 ft. 6¾ inches long, according to the ratio obtained in the preceding part of this paper, where it was found that the strength varied inversely as the 1·7 power of the length. This remark will apply to the following pillars. This casting was not very sound.
18.	Hollow uniform cylinder. External diameter 2·31 inch. Internal diameter 1·67 inch. Length 4 ft. 9 inches.	....	....	 36382	.... 1005·3	This casting was sound, and the thicknesses of the metal on opposite sides were as 10 : 21.
19.	Hollow uniform cylinder. External diameter ...... 1·85 Internal diameter by measure .... 1·35 Internal diameter by calculation .... 1·37 Mean... 1·36 Length 2 ft. 7 inches. Weight of 2 ft. 5 inches = 8 lbs. 15½ ozs.	·35	32587	33763	784·9	The thicknesses of the metal on opposite sides were as 11 : 15. In this pillar the length was 16¾ times the diameter, and the fracture took place by flexure as before; but a wedge was formed near the centre, which split the pillar at the place of fracture, and seemed to be an axis round which it turned.

Table IX.—Hollow Cylinders flat at the Ends.

Results of Experiments on the Strength of Hollow Cylinders of Cast Iron (Low Moor, No. 3.), the ends being turned flat and perpendicular to the sides, and the pressure communicated by the approach of parallel surfaces, against which the ends of the pillar were firmly bedded. Length of each pillar 7 feet 6¾ inches, except otherwise mentioned. Plate XV. No. 8, A, B, C, and D.

No. of Experiment.	Description of Pillar.	Deflection	Weight producing the deflection.	Breaking weight, or that with which it sunk.	Value of x from formula $x = \frac{W}{D^{3\cdot55} - d^{3\cdot55}}$, where W = the breaking weight, D, d the external and internal diameters; and the length of the cylinder = 7 feet 6¾ inches.	Ratio of the thicknesses of the ring of metal on opposite sides at place of fracture.	Remarks.
		inch.	lbs.	lbs.	lbs.		
1.	Hollow uniform cylinder, same as in Experiment 1. of the preceding table. inch. External diameter ...... 1·78 Internal diameter by measure ...... 1·21 Internal diameter by calculation ...... 1·21 Mean... 1·21 Length of cylinder 7 ft. 4¾ inches. Weight (length 7 ft. 6¾ ins.) 31 lbs.	·02 ·03 ·00 ·05 ·07 ·09 ·11 ·17 ·20 ·30 ·42 ·50 ·66	2813 3821 unloaded 4829 5837 6845 8861 10877 12001 14353 15921 16705 17489	17840	2973·7	1 : 5	When this cylinder was broken, it was found that the thinner side was that which was compressed. This cylinder was not reduced in its thickness toward the ends. The weight of all the cylinders below, whether their lengths are 7 feet 6¾ inches, or 7 feet 4¾ inches, are given for the greater length.
2.	Hollow uniform cylinder, same as in No. 2. preceding table. External diameter ...... 1·74 Internal diameter......... 1·187 Length of cylinder 7 ft. 4¾ inches. Weight of cylinder 7 ft. 6¾ inches = 30¼ lbs.	·12 ·17 ·32 ·48 direction changed. ·54	11217 13177 15137 15921 16313	16705	3031·5	7 : 11	This cylinder was reduced to half its thickness near to the ends, and to three-fourths of its thickness half way between the middle and the ends; but the weight of the cylinder as set down is for the whole before it was reduced. The fracture took place in the middle, and in another place near to it, but not in the reduced parts.
3.	Hollow uniform cylinder. External diameter ...... 1·76 Internal diameter by measure ...... 1·18	bent ·03 ·05 ·06 ·09 ·25 ·36 ·54	2141 2749 5437 6333 6677 13721 15233 16241	16745	2968·7	1 : 3	This cylinder was reduced in the same manner as the last, or somewhat more, and the fracture took place at the reduced part, half way between the middle and one end; where the external diameter was 1·60 and the internal 1·18, and the thickness of the metal on the opposite sides = ·09 + ·32 = ·41. But the thickness of the metal in the middle of the pillar was 1·76 − 1·18 = ·58: therefore the reduced part was somewhat less than three quarters of the whole.
4.	Hollow uniform cylinder. External diameter ...... 1·75 Internal diameter by measure ...... 1·09 Internal diameter by calculation ...... 1·14 Mean... 1·11 Weight of cylinder 32 lbs.	·01 ·05 ·06 ·08 ·10 ·12 ·15 ·19 ·25 ·31 ·36 ·42 ·46 ·55	2237 4441 6569 8165 12445 13789 15133 16477 17821 18493 19165 19613 20061 20509	20957	3586·9	1 : 2	This pillar was a good sound casting, and was not reduced in its thickness in the manner of the last two.

TABLE IX. (Continued.)

No. of Experiment.	Description of Pillar.	Deflection.	Weight producing the deflection.	Breaking weight, or that with which it sunk.	Value of x from formula $x = \frac{W}{D^{3\cdot55} - d^{3\cdot55}}$, where W= the breaking weight, D, d the external and internal diameter; and the length of the cylinder = 7 feet 6¾ inches.	Ratio of the thicknesses of the ring of metal on opposite sides at place of fracture.	Remarks.
	inches.	inch.	lbs.	lbs,	lbs.		
5.	Hollow uniform cylinder. External diameter ...... 2·04 Internal diameter by measure ...... } 1·44 Internal diameter by calculation ...... } 1·49 Mean... 1·46 Length of cylinder 7 ft. 4¾ inches. Weight of cylinder 7 ft. 6¾ inches = 35½ lbs.	·04 ·05 ·07 ·08 ·10 ·12 ·15 ·17 ·20 ·26 ·37 ·52	3589 7387 11091 14703 17391 20011 23481 25105 26729 28353 29977 31601	32413	3573·8	1 : 1	This column was not reduced in thickness as above; it was a good casting, of equal thickness on all sides. A wedge broke out at the place of fracture which was 4¾ inches from the middle, and showed that the ver sin, or depth of the part in a state of tension, was 1·72 inch; ∴ depth of compressed part = 2·04 − 1·72 = ·32 inch. See W. (Plate XV., No. 8, D).
6.	Hollow uniform cylinder. External diameter ...... 2·01 Internal diameter by measure ...... } 1·330 Internal diameter by calculation ...... } 1·407 Mean... 1·368 Length of cylinders 7 ft. 6¾ ins. Weight of cylinder 37¾ lbs.	·03 ·05 ·07 ·08 ·00 ·14 ·20 ·27 ·38 ·53	3589 5819 12091 18667 unloaded 21857 25105 26729 28353 29977	30789	3290·3	7 : 10	This cylinder had a slight bubble in the place of fracture, which it was conceived had little influence on the strength. The ends of the cylinder were not reduced.
7.	Hollow uniform cylinder, same as No. 3. of Table VIII. External diameter ...... 2·01 Internal diameter by measure ...... } 1·40 Internal diameter by calculation ...... } 1·43 Mean... 1·415 Length of cylinder 7 ft. 4¾ inches. Weight of cylinder 7 ft. 6¾ inches = 36½ lbs.	bent bent ·10 ·01 ·14 ·25	4251 5819 21857 unloaded 25917 27541	28353	3214·8	5 : 11	This cylinder was the same as that in Experiment 3. of the last Table. It was rendered quite straight, and its ends were firmly bedded against the crushing surfaces. Previous to this experiment, however, the cylinder was reduced to half the thickness near to the ends, and to three quarters of the thickness half way between the middle and the ends. In the former cases, where the cylinder was reduced, the fracture took place only in the middle, but in this, the pillar broke both in the middle and at the two reduced places half way between the middle and the ends. It was broken too at the reduced part near to one end, and the other end was split. Casting moderately sound; the thicker side of the casting was the compressed one in this instance.
8.	Hollow uniform cylinder. External diameter ...... 1·99 Internal, both by measure and calculation ...... } 1·31 Length of cylinder 7 ft. 5·8 ins. Weight of cylinder 7 ft 6¾ inches, before reduction 39 lbs.	bent ·02 ? ·20 ·26 ·33 ·45 ·55 ·90	1456 3355 15605 17755 19905 22055 24205 26731	27067	2988·3	6 : 7	This cylinder was reduced in the manner of the preceding ones. It bent upwards of an inch, and then broke; first at a small flaw near the middle, then at the reduced part half way between the middle and one end. A piece broke off at the neutral line; it was nearly straight across in a line with the inner diameter. Depth of compression = ·36 inch.

TABLE IX. (Continued.)

No. of Experiment.	Description of Pillar.	Deflection.	Weight producing the deflection.	Breaking weight, or that with which it sunk.	Value of x from formula $x = \frac{W}{D^{3\cdot55} - d^{3\cdot55}}$, where W = the breaking weight, D, d the external and internal diameters; and the length of the cylinder = 7 feet 6¾ inches.	Ratio of the thicknesses of the ring of metal on opposite sides at place of fracture.	Remarks.
		inch.	lbs.	lbs.	lbs.		
9.	Hollow uniform cylinder, same as No. 5. last table. inches. External diameter ...... 2·23 Internal diameter by measure .... } 1·53 Internal diameter by calculation .... } 1·55 Mean... 1·54 Length of cylinder 7 ft. 4¾ inches. Weight of cylinder 7ft. 6¾ inches, before reduction 47 lbs.			40569	3099·0	4 : 9	This cylinder was reduced in thickness in the same manner, and to the same degree, as before. It broke in six pieces, in the middle, at one of the reduced parts half way between the middle and the ends, where the diameter was 2·12 inches, and had both of its ends split. In this, and the preceding cases, the fracture seems to have taken place, first in the middle, and afterwards, by reaction, in the other places; but it is possible that some cracking at the ends preceded the rupture.
10.	Uniform *solid* cylinder cast in *green* (moist) sand. Diameter 1·76 inch. Length 7 feet 6¾ inches. Weight 56 lbs.	bent bent ·35 ·65	4135 10855 21219 22787	23179	3115·5		With 23179 lbs. it became bent more than an inch, and slipped out of the place; it was afterwards rendered straight and replaced; and it would have broken with a less weight.
11.	Uniform *solid* cylinder cast in *dry* sand. Diameter 1·72 inch. Length 7 feet 6¾ inches. Weight 53 lbs 8 oz.	·20 ·23 ·28 ·35 ·44 ·65	16115 17235 18355 19475 20595 21715	21995	3207·7		With the last weight, 21995, the pillar slipped from its fixings, as the preceding one had done, and when replaced it was broken with a less weight than it had borne before. The weight, in both this case and the last, was so near to what the breaking weight must have been, if the fracture had been effected as usual, that I have not hesitated to put down the results as those of fracture.

Table X.—Uniform Hollow Cylindrical Pillars of Low Moor Iron, No. 3; the ends being flat, and the lengths less than thirty times the external diameter. Plate XV. No. 8, E, F.

Number of Experiment.	Length of pillar.	External diameter.	Internal diameter.	Weight of pillar.	Breaking weight.	Value of b.	Value of c.	Calculated breaking weight from formula. $\frac{bc}{b+\frac{3c}{4}}$.	Remarks.
	inches. feet.	inch.	inch.	lbs. oz.	lbs.	lbs.	lbs.	lbs.	
1.	30·25 = 2·5208	1·26	·767	6 2	33679	38807·6	86178·5	32331	Not perfectly sound.
2.	30·25 = 2·5208	1·26	·781	6 1	32867	38274	84310	31790	Core not quite in middle, thicknesses of metal on opposite sides 3 : 4.
3.	26 = 2·1666	1·25	·768	5 2	35302	48461·7	83882	36501	Air bubbles in casting.
4.	26 = 2·1666	1·17	·752	4 7	31195	36887	69283·2	28764	Core in centre, T : C : : 43 : 74.
5.	23 = 1·91666	1·16	·7705	3 9	30383	42633	64844·7	30291	Core in centre, T : C : : 11 : 18.
6.	20·166 = 1·6805	1·21	·77	3 9	41751	64599·2	75130·6	40128	Core in centre, broke in middle in many pieces with sharp points.
7.	20·166 = 1·6805	1·14	·805	2 11	27135	46408	56193	29449	Broke in middle in twelve pieces.
8.	17 = 1·4166	1·15	·91	1 11	25511	50927	42636	26191	
9.	16 = 1·3333	1·15	·92	1 9½	25105	54730	41053	26273	Broke in middle in twenty-five pieces.
10.	15·125 = 1·2604	1·16	·932	1 8	26729	61304·1	41133·8	27364	Core in centre, T : C : : 52 : 64.
11.	15·125 = 1·2604	1·08	·77	1 12	27135	61570·2	49457	30863	
12.	14 = 1·16666	1·15	·792	2 0	37285	91909	59953	40257	
13.	8·8 = ·73333	1·13	·91	13½	34037	133000	38704	31750	Broke in middle in ten pieces.

Most of the pillars, which were under twenty inches long, were turned outside and bored within. The whole in this table were (with the exceptions named) very good, and the core was nearly in the centre of all, except the 2nd. By the ratio T : C is to be understood the depth of the part extended to that compressed in the section of fracture.

By the values of b in the seventh column is to be understood the breaking weights calculated from the formula $b = 99318 \frac{D^{3\cdot55} - d^{3\cdot55}}{l^{1\cdot7}}$, for long pillars (Art. 51.); where l is the length of the pillar, and D, d the external and internal diameters. The values of c in the eighth column are the weight which would crush, without flexure, a mass of iron of the same section as the pillar (Art. 55); and the breaking weight in the ninth column is computed by the formula (Art. 43.) for short pillars: strength $= \frac{bc}{b + \frac{3}{4}c}$. It having been shown, I hope satisfactorily, that the strength of short pillars, which break by flexure, depends upon their resistance to crushing (Art. 39–42.), I have endeavoured there to show the laws on which it depends; and the agreement between the calculated and the experimental results, in hollow pillars, may be judged of from the above results.

TABLE XI.—Pillars of various forms and different modes of placing.

No. of Experiment.	Description of pillar (all being cast from the same iron as before).	Deflection.	Weight producing the deflection.	Breaking weight.	Remarks.
		inch.	lbs.	lbs.	
1.	Round hollow pillar, regularly tapering from the middle to the ends, like the frustums of two cones, whose bases joined in the middle of the pillar. Plate XV. No. 9, C, where the external diameter was 2·24 inches, and the internal 1·63 inch. The extreme diameter near the end was 1·73 inch; the length of the pillar 7 ft. 6¾ inches, and its weight 36 lbs. It had strong discs at the ends, 2½ inches diameter, turned flat; but was cast to be everywhere else of uniform thickness.	·04 ·07 ·17 ·18 ·34 ·54	4135 10855 21219 after hanging one hour. 25139 27099	28667	It broke in five parts; 2½ and 11 inches from the middle, and near the disc at each end; the fracture seeming to have taken place first near the middle. The thickness of the metal at the place nearest to the middle was ·33+·25=·58 inch; and at the other near to it ·33+·26=·59 inch. At this last place a wedge broke out, which showed the compressed part to be $\frac{41}{120}$th of the diameter.
2.	Round hollow pillar from the same model as the last. External diameter in middle 2·24 inches. External diameter near to the ends 1·73 inch. Length of pillar 7 ft. 6¾ inches. Weight of pillar 36¼ lbs. Plate XV. No. 9, D.	bent. ·03 ·15 ·35 ·65	3603 12105 21219 25139 27099	27491	It broke eighteen inches, and twenty-five inches, from the middle at the same end, and at the disc at that end. A wedge broke out which showed the compressed part to be $\frac{54}{216}$th of the diameter. The thickness of the metal at the place of fracture nearest to the middle was ·19 + ·34 = ·53 inch; and in another place ·34 + ·25 = ·59.
3.	Round hollow pillar, regularly tapering from the bottom to the top; as the frustum of a hollow cone. Plate XV. No. 9, A or B. External diameter at bottom 2·22 inches. External diameter at top 1·70 inch. Internal diameter at bottom 1·61 inch. Length of pillar 7 ft. 6¾ inches. Weight of pillar 39 lbs. Its ends were turned flat, and it had a strong disc upon the top of double the diameter there, all the rest was cast to be of uniform thickness.	bent. ·02 ·12 ·22 ·30 ·42 ·60 ·87	3355 12403 21857 25917 28353 29977 31601 32413	32413	It broke about two inches below the middle, with the last weight, after bearing it about a minute. Thickness of metal there ·25 + ·36 = ·61 inch. The disc at the top of this pillar was intended to give it a firmer bedding than it could otherwise have had from its small diameter.
4.	Round hollow pillar of the same form as the last, but from a different model. External diameter at bottom 2·32 inches. External diameter near top 1·75 inch. Diameter of disc at top of pillar 3 inches, its thickness being ·9 inch. Length of pillar 7 ft. 6¾ inches. Weight of pillar 34 lbs. 9 ozs. The pillar, excepting the disc, was cast to be everywhere ¼ inch thick. Plate XV. No. 9, A or B.	·06 ·09 ·20 } direction altered a little. ·28 ·45 ·62 ·80	6617 12105 17593 20729 22027 24043 25387	26563	The fracture took place 2 ft. 11½ inches from the bottom, where the external diameter was 2·08 inches, and the internal 1·57 inch. The thickness of metal on the opposite sides there was ·35 + ·15 = ·50 inch. The thin side was, as often happened, the compressed one; and a small wedge broke out at the place of fracture, showing that two-thirds of the diameter at least was in a state of tension. There was a defect called "cold shot" about one-eighth of an inch in diameter in the tensile part of the fracture, which would reduce the strength a little.
5.	Uniform pillar, whose section was of the form in Plate XV. No. 6, B, where a b = c d = 3 inches, and the thickness of the ribs = ·48 inch. Length of pillar 7 ft. 6¾ inches. Weight of pillar 62 lbs. The ends were rounded, that the force might pass through the axis.	·04 ·07 ·14 ·25 ·45	11169 13129 15089 16100 17175	17578	It sunk down, bending in the direction of one of the ribs. This experiment was made to give some idea of the strength of a connecting rod of the usual form, as compared with that of a hollow cylinder of the same weight and length.
6.	Uniform pillars, whose section was cast to be equal in area to that in experiment 5, but of the form in Plate XV. No. 6, A, in which a b = c d = 3 inches, e f = 2½ inches. Length of pillar 7 ft. 6¾ inches. Weight of pillar 63½ lbs. Ends rounded.	·07 ·13 ·25	19863 23895 25507	29571	It broke by bending in the direction a b, c d, and was but little bent in the direction e f.
7.	Solid uniform cylinder, formed at the ends so that the strain would not pass through the axis, but in a line half way between the centre and the circumference. Diameter of pillar 1·95 inch. Length of pillar 60·5 inches. Weight of pillar 45 lbs. 14 ozs. Plate XV. No. 5, B.	·96* 1·30	11011 12691	13195	*The deflections here given are not those from the whole length of the pillar, as in all other cases, but from a length of 4 ft. 8½ inches of the middle of it.

Table XI. (Continued.)

No. of Experiment.	Description of pillar (all being cast from the same iron as before).	Deflection.	Weight producing the deflection.	Breaking weight.	Remarks.
		inch.	lbs.	lbs.	
8.	Solid uniform cylinder, differing from the last only in the ends being formed so that the strain passed in a line further from the centre, or $\frac{1}{8}$th of the diameter from one side. Diameter of cylinder 1·96 inch. Length of pillar 60·5 inches. Weight of pillar 46 lbs. 5 ozs.	·36 ·47 ·54 ·79 ·92 1·15 1·38 1·48	7341 8685 9893 11169 11953 12737 13521 14305	14697	The deflections of this pillar were for a length 4 ft. 8$\frac{1}{2}$ inches, as in the last case.
9.	Solid uniform cylinder fixed so that the pressure passed diagonally through the opposite corners. Diameter of pillar ·97 inch. Length of pillar 30·25 inches. Weight of pillar 5 lbs. 13 ozs. Plate XV. No. 5, A.			5750	
10.	Cylinder, same as the last and broke in the same manner. Length of pillar 30$\frac{1}{4}$ inches. Diameter of pillar 1·02 inch. Weight of pillar 6 . 4$\frac{1}{4}$ ozs.			7275	Taking a mean from these two experiments, we have a cylinder, diameter ·995 inch, length 30$\frac{1}{4}$ inches, weighing 6 lbs. $\frac{5}{8}$ oz., broke with 6512 lbs.

TABLE XII.—Experiments on the Resistance of Pillars of the best Staffordshire Wrought Iron and Steel.

Description of Pillar.	Length of pillar.	Diameter of pillar.	Mean diameter of pillar.	Weight of pillar.	Mean weight of pillar.	Deflection.	Weight causing the deflection.	Breaking weight in lbs.	Mean breaking weight.	Remarks.
1st. WROUGHT IRON.	ft. ins.	inch.	inch.	lbs oz.	lbs. oz.	inch.	lbs.	lbs.	lbs.	
Solid uniform cylinder, both ends rounded Plate XV. No. 1, A.	7 $6\frac{3}{4}$	1·02	1·017	20 $4\frac{1}{2}$	20 $3\frac{1}{4}$	bent. ·03 ·04 ·05 ·12 ·42	296 352 800 1360 1640 1780	1825	1808	
Solid uniform cylinder, both ends rounded...	7 $6\frac{3}{4}$	1·015		20 2		bent. ·03 ·04 ·11 ·48	240 352 800 1360 1720	1792		The results here given are the means from two experiments. The ends of the pillar were not perceptibly flattened by the pressure.
Solid uniform cylinder, both ends rounded...	5 $0\frac{1}{2}$	1·015	1·015	13 0	12 $15\frac{3}{4}$	·07	2804	3812	3938	The ends of the pillars were little or none crushed, or flattened by the pressure.
Solid uniform cylinder, both ends rounded...	5 $0\frac{1}{2}$	1·015		12 $15\frac{1}{2}$		·10	3308	4064		
Solid uniform cylinder, both ends rounded...	2 $6\frac{1}{4}$	1·015	1·015	6 12	6 12	·0 ·01 ·02 ·05 ·06 ·06	2237 4537 8121 12181 13525 14869	14869	15480	In this pillar and the next the ends were rendered flat by the pressure; the diameter of the flattened part in each was ·4 inch.
Solid uniform cylinder, both ends rounded...	2 $6\frac{1}{4}$	1·015		6 12				16213		
Solid uniform cylinder, both ends rounded...	2 $6\frac{1}{4}$	1·015		6 12				15357		This pillar had the ends tipt with hardened steel to prevent it being crushed.
Solid uniform cylinder, both ends rounded...	2 $6\frac{1}{4}$	·52	·52	1 13	1 13	·0? ·05	742 1246	1330	1260	Ends little or none crushed.
Solid uniform cylinder, both ends rounded...	2 $6\frac{1}{4}$	·52		1 13		·17	1078	1162		
Solid uniform cylinder, both ends rounded...	2 $6\frac{1}{4}$	·52		1 13		·10 ·24	1078 1246	1288		
Solid uniform cylinder, both ends rounded...	1 $8\frac{1}{6}$	1·015	1·015					23312	20834	The ends of this cylinder were somewhat crushed and flattened.
Solid uniform cylinder, both ends rounded...	1 $8\frac{1}{6}$	1·015						18356		This cylinder had its ends tipt with hardened steel to prevent them becoming flattened.
Solid uniform cylinder, both ends rounded...	1 $3\frac{1}{8}$	1·005	1·005	3 $5\frac{1}{2}$	3 $5\frac{1}{2}$			23395	23535	Ends rendered flat by the pressure to $\frac{1}{2}$ the diameter, or $\frac{1}{4}$ the area.
Solid uniform cylinder, both ends rounded...	1 $3\frac{1}{8}$	1·005		3 $5\frac{1}{2}$				23675		The ends of this were tipt with steel, but as it was not hardened, the ends became flattened, though in a less degree than in the last instance.
Solid uniform cylinder, rounded at one end and flat at the other. Plate XV. No. 1, B.	7 $6\frac{3}{4}$	1·02	1·02	20 5	20 5	bent. ·02 ·07 ·15 ·27	1080 1920 2480 3040 3320	3355	3355	In this pillar the greatest flexure was about 2 feet $11\frac{1}{2}$ inches from the rounded end.
Solid uniform cylinder, rounded at one end and flat at the other.	5 $0\frac{1}{2}$	1·03	1·03	13 12				8137	8137	The rounded end was much flattened by the pressure in the shorter pillar, and slightly so in the longer, which had left a flat part at the end ·28 inch diameter.
Solid uniform cylinder, rounded at one end and flat at the other.	2 $6\frac{1}{4}$	1·015	1·015	6 13	6 13			21355	21355	

Table XII. (Continued.)

Description of Pillar.	Length of pillar.	Diameter of pillar.	Mean diameter of pillar.	Weight of pillar.	Mean weight of pillar.	Deflection.	Weight causing the deflection.	Breaking weight.	Mean breaking weight.	Remarks.
1st. WROUGHT IRON.	ft. ins.	inch.	inch.	lbs. oz.	lbs. oz.	inch.	lbs.	lbs.	lbs.	
Solid uniform cylinder, one end rounded and a disc on the other. Plate XV. No. 2, B.	2 6¼	1·015	1·015	7 2				22699	21187	Rounded and flattened by the pressure.
Solid uniform cylinder, one end rounded and a disc on the other.	2 6¼	1·015						19675		The rounded end of this was tipt with hardened steel to prevent crushing. The greatest deflection was about 11¼ inches from the rounded end.
Solid uniform cylinder, rounded at one end and flat at the other. Plate XV. No. 1, B.	1 3⅛	1·005	1·005	3 5½	3 5½			28075	26227	It became crushed at the rounded end, so that the flattened part was ·55 inch diameter. The greatest deflection was about 7 inches from rounded end, or ½ inch from middle.
Solid uniform cylinder, rounded at one end and flat at the other.	1 3⅛	1·005		3 5½				24379		This had the rounded end tipt with hardened steel.
Solid uniform cylinder, flat at both ends ... Plate XV. No. 1, C.	7 6¾	1·02	1·02	20 6	20 6	·03 ·07 ·11 ·14 ·15 ·19 ·22 ·28 ·37 ·43	324 800 1360 1920 2480 3040 3600 4160 4720 5000	5280	5280	
Solid uniform cylinder, flat at both ends ...	5 0½	1·02	1·02	13 11	13 11			12990	12990	
Solid uniform cylinder, flat at both ends ...	2 6¼	1·015	1·015	7 1½	7 1½	·05	22027	23371	23371	
Solid uniform cylinder, with disc at both ends. Plate XV. No. 2, C.	2 6¼	1·015	1·015	7 10½	7 10½	·03 ·04	22027 25387	25387	25387	It sunk with this weight after bearing it some minutes.
Solid uniform cylinder, flat at both ends. Plate XV. No. 1, C.	1 3⅛	1·005	1·005	3 7½	3 7½			27099	27099	
Solid uniform cylinder, flat at both ends ...	1 3⅛	1·005		3 7½				27099		
2nd. CAST STEEL. NOT HARDENED.										
Solid uniform cylinder, rounded at both ends. Plate XV. No. 1, A.	29·95	·87		5 2½				10516	10516	It bore this weight some minutes, and then sunk gradually by bending; ends slightly flattened by pressure.
Solid uniform cylinder, one end rounded and the other flat. Plate XV. No. 1, B.........	29·95	·87		5 3				20315	20315	Flattened a little at the rounded end; greatest curvature between ⅖ and ⅓ of the length from the rounded end.
Solid uniform cylinder, both ends flat. Plate XV. No. 1, C..........	29·95	·87		5 3½		·07 ·10 ·11	18667 22027 25387	26059	26059	It bore this five minutes, and then sunk; the ends were not split, as frequently happened, in a slight degree, both in cast and wrought iron.

Dantzic Oak.

TABLE XIII.—Square Pillars of Dantzic Oak, from a very good plank, which had been cut up about nine months.

Description of Pillar.	Length of pillar.	Side of square.	Weight of pillar.	Deflection.	Corresponding weight.	Breaking weight.	Mean breaking weight.	Remarks.
	inches.	inch.	lbs. oz.	inch.	lbs.	lbs.	lbs.	
Uniform pillar rounded at the ends that the force might pass through the axis. Plate XV. No. 3, A. ...	60·5	1·75		·09 ·17	2237 3197	3645	3197	Broke anglewise; it was slightly flattened at the ends by the pressure.
Uniform pillar rounded at the ends that the force might pass through the axis	60·5	1·75		·13	2141	2749		This was capped at the ends with iron to prevent them being crushed with the pressure. It bent and broke diagonally.
Uniform pillar, one end rounded and the other flat. Plate XV. No. 3, D.	60·5	1·75		·09 ·11 ·13 ·16 ·20 ·27 ·48	2141 3197 3645 4541 5437 6333 7229	7229	6109	The rounded end was much crushed.
Uniform pillar, one end rounded and the other flat...........................	60·5	1·75		·06 ·08 ·11 ·15 ·17 ·19 ·23 ·32	1070 1598 2270 2718 3166 3614 4093 4541	4989		In this pillar the rounded end was capped with iron to prevent the end being crushed.
Uniform pillar, both ends flat. Plate XV. No. 3, E.	60·5	1·75	4 10½	·02 ·04 ·14	3355 4795 9499	10171		It was crushed at the end diagonally through the centre, and sunk down by flexure. No. 3, F.
Uniform pillar, both ends flat. Plate XV. No. 3, E.	60·5	1·75				11179		This sunk by bending in the middle as usual; a portion of both ends was cracked.
Uniform pillar, both ends flat. Plate XV. No. 3, E.	60·5	1·75		 ·05 ·09 ·11	3211 5467 6139 1070	8323	9625	With 8323 lbs. it sunk down by bending, and when unloaded had taken a permanent set of ·52 inch.
Uniform pillar, both ends flat. Plate XV. No. 3, E.	60·5	1·75		·03 direction altered. ·02 ·04 ·07 ·14	3390 4459 6811 8155 8827	8827		This deflection was taken along the side, but it sunk down diagonally, one end was slightly crushed. Most of these pillars changed the direction of flexure as they became loaded.
Uniform pillar, flat at the ends, and well-bedded	29·75	1·75	2 7½			13083		It was crushed at the ends with the pressure, which caused it to break with a less weight than otherwise.
Uniform pillar, flat at the ends, and well-bedded	30·25	1·75				14305		With less weight than this, no cracking at the ends took place; but with this, it wrinkled at the ends, bent and sunk down.
Uniform pillar, flat at the ends, and well-bedded	48	1·75				9229		It was slightly bent before the weight was laid on; no cracking at the ends perceived before fracture by bending.
Uniform pillar, flat at the ends............ Uniform pillar, flat at the ends............ Uniform pillar, flat at the ends............	46·1 46·1 46·1	1·02 1·02 1·02	1 2½			1791 1791 1679	1754	These generally bent diagonally without crushing at the ends.
Uniform pillar, flat at the ends............ Uniform pillar, flat at the ends............ Uniform pillar, flat at the ends..........	46·1 46·1 46·1	1·50 1·50 1·50	2 12½			8069 8049 7545	7888	In the first and second pillar there was no cracking at the ends previous to fracture. The last was slightly split at one end by drying, and failed there.

Note.—By breaking weight above, is understood that which overcame the resistance of the pillar, and with which it sunk down.

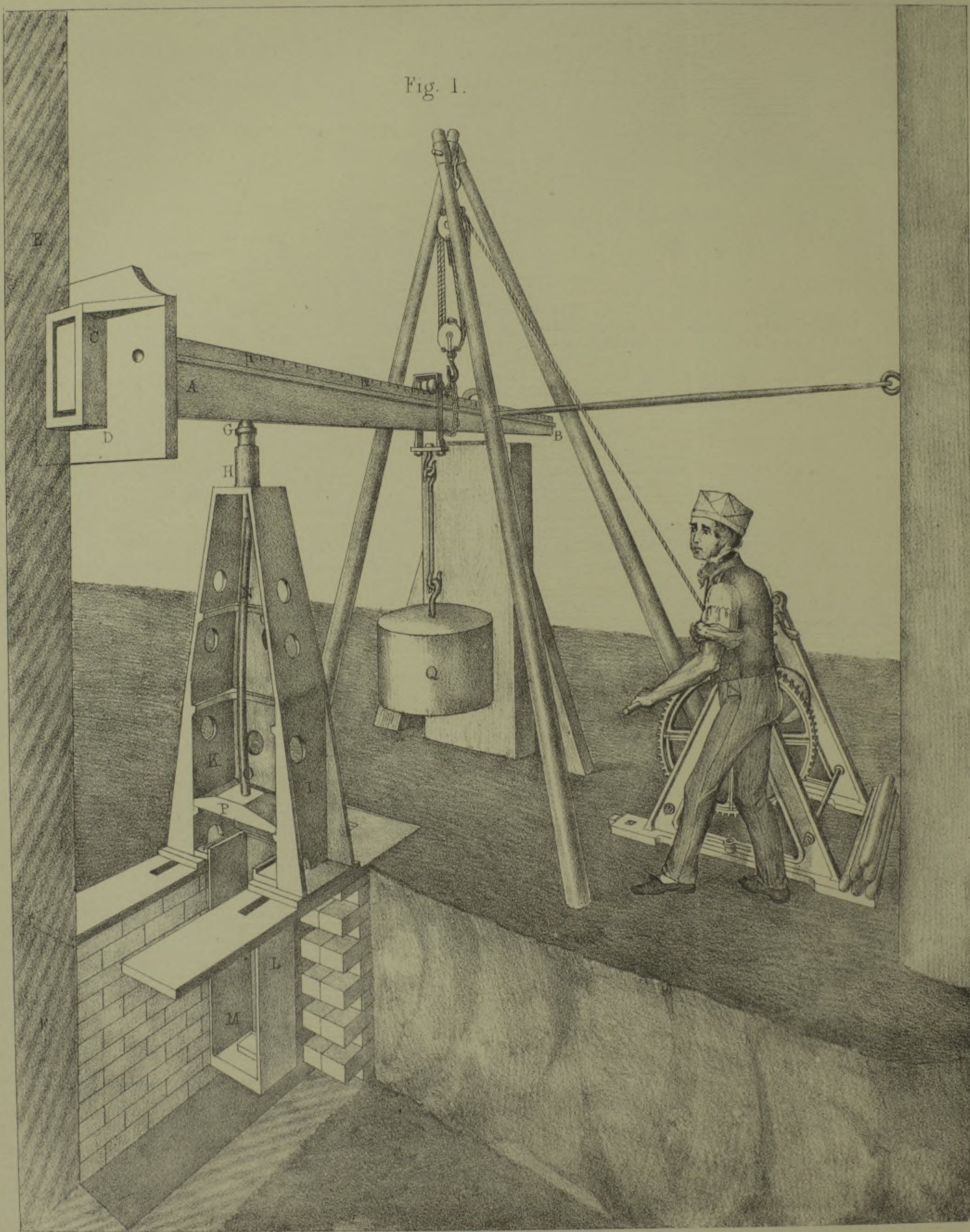
Table XIII. (Continued.)

Red Deal.

Uniform Rectangular Pillars of Red Deal, flat at the ends, cut out of the same plank; each pillar being four feet ten inches long, and having the same weight, 3 lbs. 13 ozs., and nearly the same area of section, four square inches. The sections were as in Plate XV. No. 10.

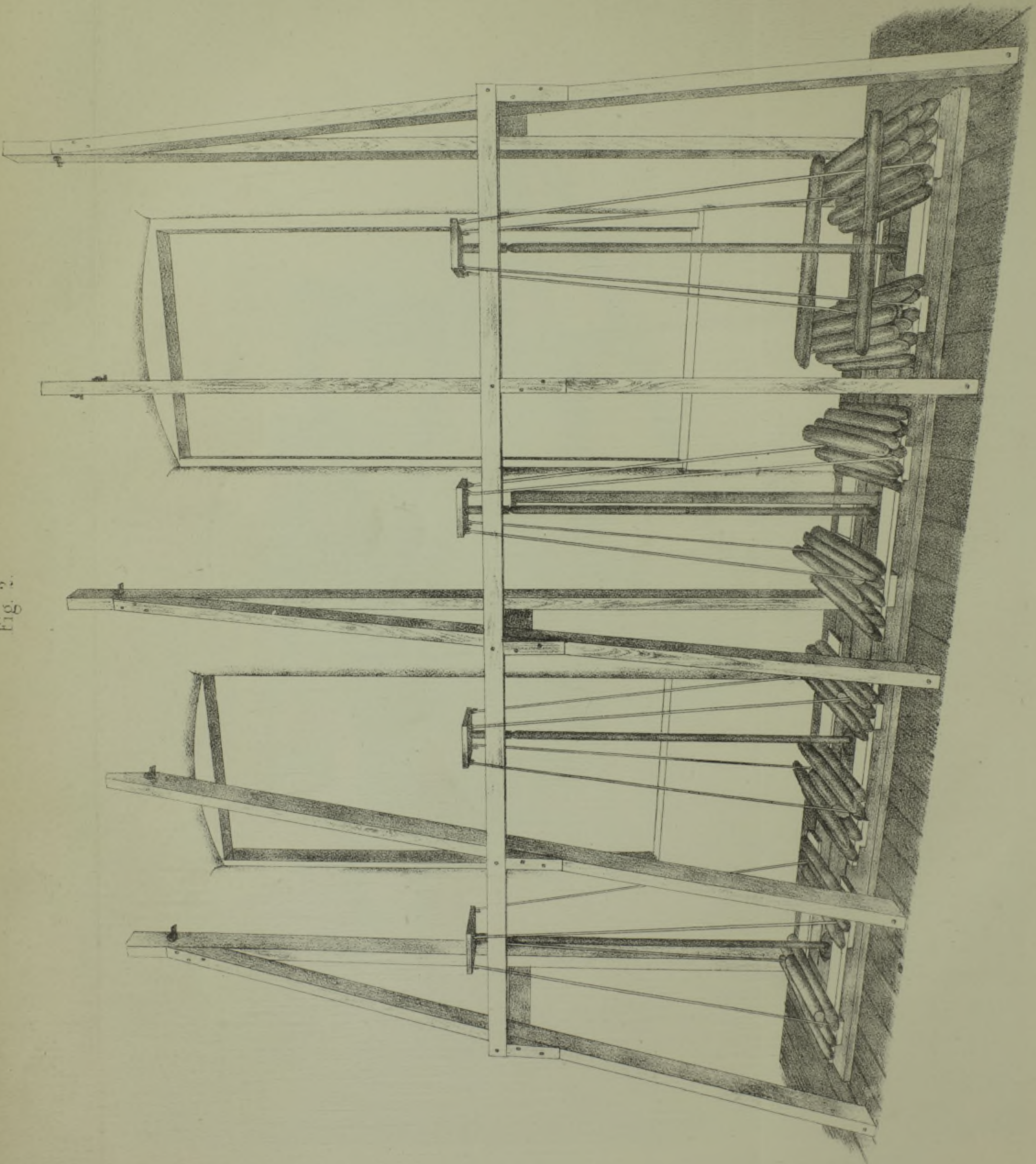
No. of Experiment.	Ratio of sides of pillar.	Deflection.	Corresponding weight.	Weight with which it sunk down.	Remarks.
		inch.	lbs.	lbs.	
1.	1 to 1	·03	3254		Bent in direction of diagonal, one end crushed about one inch from bottom; and in the middle, two-thirds or three-fourths of the area being crippled.
		·10	6505		
		·12	8857		
		·17	11209	12385	
2.	1 to 1	·09	4153		
		·12	6505	Mean 11993	
		·15	8857		Crushed as before.
		·32	11209	11601	
3.	1 to 2	·13	4153		Sunk by bending in the direction of the smaller side.
		·17	5329		
		·25	6505	7681	In another experiment, a piece of the same size broke with somewhat less than this.
4.	1 to 3	·42	4153	4349	Sunk by bending in the direction of the smaller side.

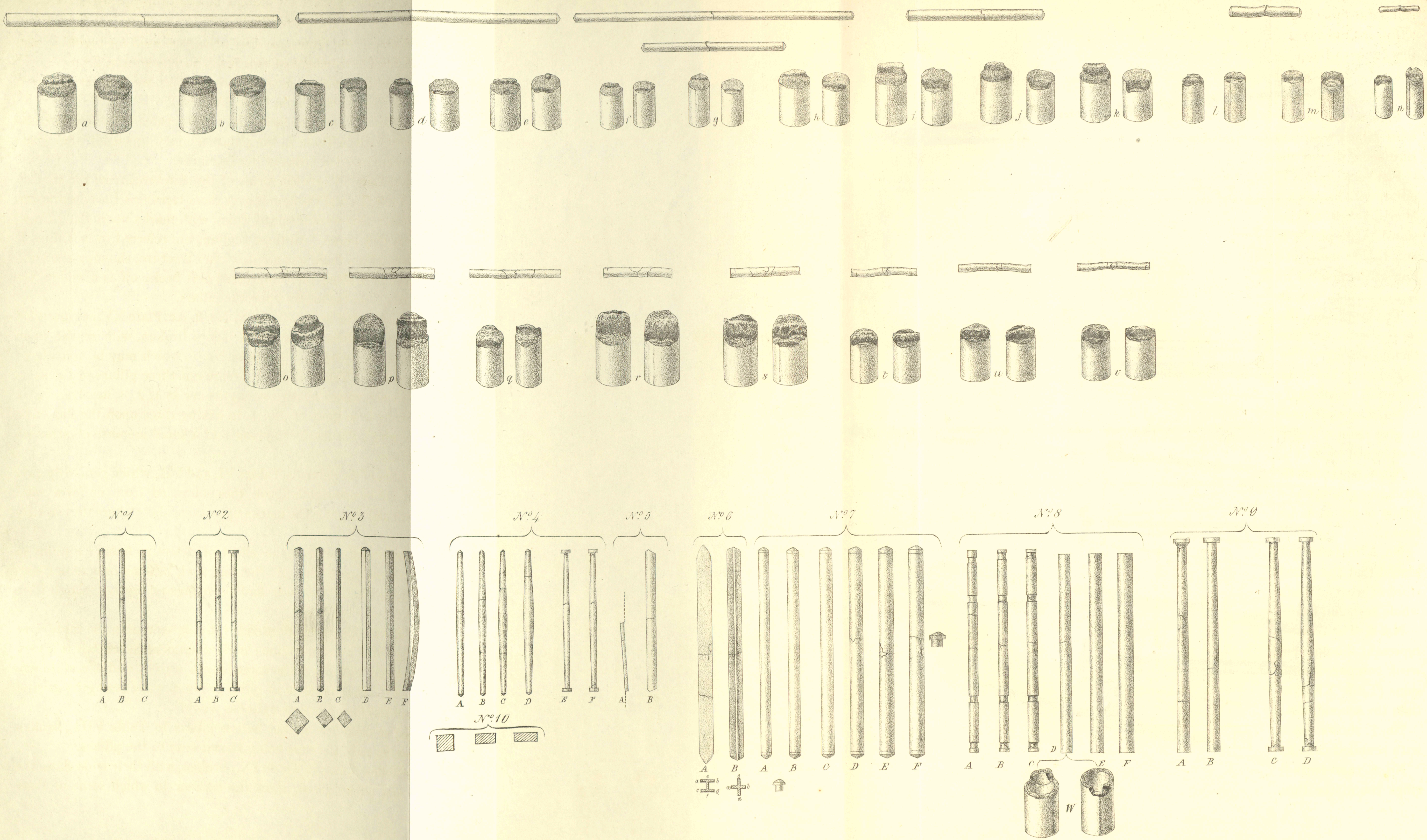
Fig. 1.



J. Basire lith.

Fig. 2.





J. Basire, lith.

Explanation of the Plates.

Fig. 1. Plate XIII. represents the lever and other apparatus used for breaking the pillars: it has been described in the commencement of this paper (Art. 2.).

Fig. 2. Plate XIV. shows the manner in which the pillars were subjected to a constant strain, to try the effect of time upon them (Art. 53.).

Figs. *a*, *b*, *c*, &c. to *n*, Plate XV. are the forms of the fractures from some of the solid pillars with rounded ends in Table I., the pillars as they appeared after fracture being drawn and placed over the forms of the fractures.

Figs. *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, Plate XV. are the forms of the fractures from the shorter pillars with flat ends in Table II.; and the figures over them represent the appearances after fracture of the pillars they were obtained from, with marks upon the pillars, showing the places where they broke. All these sections are referred to in Tables I. and II. In many of these pillars there was a crack after fracture, showing the position of the neutral line; and in some, the compressed part broke off as a wedge, the form of which may be seen from the marks upon the pillars.

The groups of pillars, designated as No. 1, No. 2, No. 3, &c. Plate XV. are intended to represent the forms of the different cast-iron pillars broken, with marks upon them, showing where they usually broke. Those in No. 1, which may be considered as including the longer pillars in Tables I., II., V., represent three pillars of the same material, length and diameter, whose relative strengths are as 1, 2, 3, nearly.

No. 2. are pillars differing from those of No. 1, in having discs upon the flat ends, to give them a larger bearing; the results showing that a small increase of strength is obtained by that addition.

No. 4. represents the forms of the pillars in Tables VI. and VII., which generally gave strengths about one-seventh or one-eighth above those obtained from uniform solid pillars of the same length and weight. The marks upon them show where they usually broke.

No. 5. includes pillars formed to show the effect of defective fixing; a pillar so placed that the pressure would pass through the diagonal, bearing only about one-third of what it would have done if the ends had been pressed upon through their whole surface (Table XI.).

No. 6. The second pillar in this group is intended, by its section in the middle, to represent the connecting rod of a steam-engine; and experiment shows that it is very weak compared with a hollow cylinder of the same length, weight, and lateral dimensions (Table XI., Art. 48.). The first form of pillar in No. 6. gave strengths greater than the second, but still less than a hollow cylinder would.

No. 7. shows the relative diameters of the hollow pillars in Table VIII., broken with rounded caps upon the ends, the caps being shown near to the pillars.

No. 8. represents the hollow pillars in Table IX., broken, with their ends flat; the rings round the three first pillars given, show the manner in which some of these

were reduced, one half of the thickness of the metal being taken away from the pillar near to the ends, and one-fourth, half way between the middle and the ends. When no more metal than this was taken away, the pillars were never broken in the reduced parts. The fourth pillar given in this group was a very good casting, and the section is represented by W below it. The small piece where the crack is seen, broke out, showing very distinctly the position of the neutral line.

The pillars No. 9. are the first four in Table XI. They were hollow, and the metal in them was cast to be all of equal thickness, except that in the discs. The marks upon them will show the places in which they broke. These pillars were somewhat weaker than uniform hollow cylindrical ones of the same length, thickness and quantity of metal (Art. 48.).

The two last pillars in No. 3. were of oak (Table XIII.); and the second of these is intended to show how timber, when much compressed, becomes wrinkled near the ends.

In this Plate the pillars are not drawn of their real lengths, but many of them are of half the length compared with the diameter.